KABARAK UNIVERSITY

EXAMINATIONS

2008/2009 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE

COURSE CODE: MATH 211

COURSE TITLE: LINEAR ALGEBRA I

STREAM: SESSION III

DAY: MONDAY

TIME: 2.00 – 4.00 P.M.

DATE: 06/04/2009

INSTRUCTIONS:

Answer question ONE and any other TWO Questions.

PLEASE TURN OVER

QUESTION ONE (30 MARKS)

(a) (i) Given $\underline{a} = 3\underline{i} + 2\underline{j} + 4k$ and $\underline{b} = -2\underline{i} + 4\underline{j} - \underline{k}$, determine $\underline{a} \times \underline{b}$ **(3 mks)**

(ii) Determine the value of λ for which the vectors $\underline{a} = 4\lambda\,\underline{i} + \lambda\,\underline{j} - \underline{k}$ and $\underline{b} = \lambda\underline{i} + 2\underline{j} + 2\underline{k}$ are orthogonal. **(3 mks)**

(b) (i) Find a linear transformation T for $\Re^2$ into the plane

$$W = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} : 2x - y + 3z = 0 \right\}$$

(6 mks)

(ii) What is meant by T being linear? **(2 mks)**

(c) Write reduced row echelon form of the system of equations below and hence determine the solution of the system. **(4 mks)**

$$x + y - z = 7$$
$$4x - y - 5z = 4$$
$$2x + 2y - 3z = 0$$

d) Let $W = \{w_1, w_2, \text{—}, w_n\}$ be a non empty subset of the vector space $\Re^n$.
Explain clearly what is meant by

(i) W is a subspace of $\Re^n$ **(2 mks)**

(ii) W is a basis for $\Re^n$ **(2 mks)**

e) (i) Derive the formula for the area of a parallelogram whose adjacent sides are $|\underline{u}|$ and $|\underline{v}|$ **(4 mks)**

(ii) If $\underline{u}$ and $\underline{v}$ are non zero vectors in $\Re^n$. Show that $|\underline{u}| + |\underline{v}| \leq |\underline{u}| + |\underline{v}|$

(4 mks)

QUESTION TWO (20 MKS)

(a) Given that V and W are two vector subspaces of a vector space U over a field F

(i) Prove that V f W is a vector subspace of U **(4 mks)**

(ii) If V = $\left\{ (x, y, z): \ x + y - 3z = 0 \right\}$ and W = $\left\{ (x, y, z): 2x + y + z = 0 \right\}$

determine the subspace V f W and find a vector S which spans this subspace. **(4 mks)**

(b) Let W be the subspace of $\Re^4$ spanned by the set U =

$\left\{ (1, 2, 1, 1), (0, 1, -1, 1), (1, 0, 2, 3), (1, -1, 2, 6) \right\}$

(i) Determine whether U is a linearly independent set of not. **(4 mks)**

(ii) Find a subject of U that forms a basis for W. **(2 mks)**

(iii) State the dimension of W. **(2 mks)**

(b) Find a basis for the nulls pace of A = $\begin{bmatrix} 3 & 4 & -1 & 1 \\ 1 & -1 & 3 & 1 \\ 4 & -3 & 11 & 2 \end{bmatrix}$ **(4 mks)**

QUESTION THREE (20 MARKS)

(a) If T: V $\rightarrow$ W is a linear transformation of a vectors space V into a vector space W,

show that the range T $\left\{ y \in W: y = T(x) \text{ for some } x \in V \right\}$ is a subspace of W **(6 mks)**

(b) A linear mapping T: $R^3 \rightarrow R^3$ is defined by

$$T([x, y, z]) = [2x + 3z, 3y + 2z, 2x + 5y]$$

(i) Find a matrix M_T that represents T with respect to the standard ordered basis for $\Re^3$ **(1 mk)**

(ii) Define the vector $\underline{a}$ if $\underline{a}$ is in the kernel of T. **(4 mks)**

(iii) Determine the vectors which span the range of T. **(4 mks)**

(iv) Determine the nullity of M_T **(2 mks)**

(c) Let T: $\Re^2 \rightarrow \Re^3$ be a linear transformation such that

$$T\left(\begin{bmatrix}1\\0\end{bmatrix}\right)=\begin{bmatrix}1\\2\\3\end{bmatrix},\ T\left(\begin{bmatrix}0\\1\end{bmatrix}\right)=\begin{bmatrix}-4\\0\\5\end{bmatrix},\ \text{find } T\left(\begin{bmatrix}x\\y\end{bmatrix}\right)$$ **(3 mks)**

QUESTION FOUR (20 MARKS)

(a) Given $V_{1 = (1, 4, 6)}$, V_2 = (1, -1, 1) and V_3 = (-1, 5, 3) determine if (7, 4, 20) is in Sp (v_1, v_2, v_3). **(4 mks)**

(b) Assume that P is a particular solution to the matrix equation A$\underline{x}$ = b and let N be the nulls pace of A. Show that the entire solution set to this equation is

$(\underline{p} + \text{n}: n \in N)$ **(5 mks)**

(c) Consider the matrix A = $\begin{pmatrix}-1 & 3 & 2\\0 & 1 & 1\\2 & -2 & 0\end{pmatrix}$

(i) Find the nulls pace of A. **(4 mks)**

(ii) Write the general solution to the equation Ax = $\begin{bmatrix}-2\\0\\4\end{bmatrix}$ in the form of a particular solution plus an arbitrary member of the nulls pace of A. **(5 mks)**

(d) Distinguish between a whole subspace and a proper subspace. **(2 mks)**

QUESTION FIVE (20 MKS)

(a) The planes x + 2y + 3z = 2 and 2x + 3y + 2z = 4 intersect along line L. Find the parametric equation of L and state its direction vector **(4 mks)**

(c) Three consecutive vertices of a parallelogram are A (2, -1, 1), B (3, 2, -1) and C (-1, 3, 2). Determine the equation of the plane in which this parallelogram lies. **(6 mks)**

(c) State and prove the Schwarz inequality. **(6 mks)**

(d) Determine the angle between the vectors
$\underline{a} = 2\underline{i} - \underline{j} + 6\underline{k}$ and $\underline{b} = 3\underline{i} + 2j + k$ **(4 mks)**