KABARAK UNIVERSITY

UNIVERSITY EXAMINATIONS

2009/2010 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE

COURSE CODE: **MATH 220**

COURSE TITLE: **LINEAR ALGEBRA II**

STREAM: **SESSION II**

DAY: **FRIDAY**

TIME: **2.00 – 4.00 P.M.**

DATE: **09/04/2010**

INSTRUCTIONS:

Attempt question **ONE** and any other **TWO** Questions

PLEASE TURN OVER

QUESTION ONE (30 MARKS)

(a) Let $T: R^2 \rightarrow R^3$ be a linear transformation such that

$$T(1) = 1,\ T(x) = x^2,\ \text{and}\ T(x^2) = x^3 + x$$

Determine;

(i) $T(3x^2 - 5x + 2)$ **(3 marks)**

(ii) $T(ax^2 + bx + c)$ **(3 marks)**

(b) On $C[a, b]$ for $b > a$, define $(f, g) = \int_a^b f(x)g(x)dx$. Show that $(f, g) = \int_a^b f(x)g(x)dx$ is an inner product. **(4 marks)**

(c) Solve the following systems of equation using row reduction method.

$$x + 3y + 6z = 25$$

$$2x + 7y + 14z = 58$$

$$2y + 5z = 19$$

(6 marks)

(d) Show that the following matix is diagonalizable and hence find a diagonal matrix similar to the given matrix; $\begin{bmatrix} 2 & 0 & 0 \\ 1 & -1 & -2 \\ -1 & 0 & 1 \end{bmatrix}$ **(6 marks)**

(e) Find the eigen values and hence the eigenvectors of the following 3 x 3 matrix.

$$\begin{bmatrix} 2 & 0 & 0 \\ 3 & -1 & 0 \\ 0 & 4 & 3 \end{bmatrix}$$

(8 marks)

QUESTION TWO (20 MARKS)

(a) Let V be an inner product space and let x and y be vectors in V. Prove that;

$$\|(x, y)\| \leq \|x\|\,\|y\|$$

(8 marks)

(b) Show that the product on $m_{2\times 2}$ defined by $(A, B) = a_{11}b_{11} + a_{12}b_{12} + a_{21}b_{21} + a_{22}b_{22}$ is inner product where

$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ $\quad B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$ **(4 marks)**

(c) Compute (A, B) for $A = \begin{pmatrix} -1 & 3 \\ 7 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 0 \\ 6 & 5 \end{pmatrix}$ using the inner product defined in (b) above. **(3 marks)**

(d) Show that $\begin{pmatrix} 2 & 5 \\ 5 & 2 \end{pmatrix}$ and $\begin{pmatrix} 0 & -3 \\ 3 & 0 \end{pmatrix}$ are orthogonal with respect to the inner product defined in (b) above. **(5 marks)**

QUESTION THREE (20 MARKS)

(a) State the condition that a matrix must have for it to be diagonalizable? **(2 marks)**

(b) For each of the following matrices state if it is diagonalizable or not.

(i) $\begin{pmatrix} 2 & -1 \\ 1 & 0 \end{pmatrix}$

(ii) $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$

(iii) $\begin{pmatrix} 3 & 4 \\ -1 & 2 \end{pmatrix}$

(iv) $\begin{pmatrix} -1 & 3 \\ 7 & 1 \end{pmatrix}$ **(8 marks)**

(c) Find an invertible matrix p for the following matrix

$$A = \begin{bmatrix} 2 & 2 & -2 \\ 1 & 3 & -1 \\ -1 & 1 & 1 \end{bmatrix}$$ **(10 marks)**

QUESTION FOUR (20 MARKS)

(a) State the Cauchy-Schwartz inequality. **(2 marks)**

(b) Verify the Cauchy-Schwartz inequality for the vectors $X_1 = (-4, 3, 0, 12)$ and $X_2 = (-2, 0, 1, 2)$ in R^4 with respect to the dot product. **(4 marks)**

(c) Explain the term an orthonormal basis for an inner product space. **(2 marks)**

(d) Show that $B = \left\{\left(\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\right), \left(\frac{-2}{3}, \frac{2}{3}, \frac{1}{3}\right)\right\}$ is an orthonormal basis for the inner product space $W =: \{(x, y, z) \in R^3 : x + 2y - 2z = 0\}$ with respect to the dot product.

(4 marks)

(e) Find an orthonormal basis for the subspace $W = \{(3t, -4t, 12t) : t \in R\}$ of R^3 with respect to the dot product. **(3 marks)**

(f) Show that the function T with domain the set of 2 x 1 vectors and defined by

$T\left[\begin{pmatrix} x \\ y \end{pmatrix}\right] = x$ is linear. **(5 marks)**

QUESTION FIVE (20 MARKS)

(a) Consider the set defined by;

$V =: \{(x, y, z)/ax + by + cz = 0, a, b, c \in R\}$

Check if V is a vector space. **(5 marks)**

(b) Let $V = \{v_1, v_2, v_3 - - - v_n\}$ be a vector in R^3. Explain clearly what is meant by V is linearly independent. **(2 marks)**

(c) Let V be a vector space in R^3 and Let $V_1 = (1, 2, 1)$, $V_2 = (1, 0, 2)$ and $V_3 = (1, 1, 0)$ Does V_1 V_2 V_3 span V? **(3 marks)**

(d) Solve the following system of equation:

$$x_1 + 2x_2 - 3x_3 = 0$$

$$2x_1 - 2x_2 - x_3 = -1$$

$$-3x_1 + 5x_2 + x_3 = 3$$

(5 marks)

(e) Consider the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$, If A is a matrix of a linear transformation, compute

(i) Ker(A) **(3 marks)**

(ii) Nullity of A **(2 marks)**