KABARAK UNIVERSITY

UNIVERSITY EXAMINATIONS

2010/2011 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF SCIENCE IN ECONOMICS AND MATHEMATICS

COURSE CODE: MATH 220

COURSE TITLE: LINEAR ALGEBRA II

STREAM: Y2S2

DAY: WEDNESDAY

TIME: 9.00 – 11.00 A.M.

DATE: 08/12/2010

INSTRUCTIONS:

1. Question **ONE** is compulsory.
2. Attempt question **ONE** and any other **TWO**

PLEASE TURNOVER

QUESTION ONE (30MKS)

(a) If u and v are any two vectors in an inner product space defined as

$$< u, v > = 2u_1v_1 + u_2v_2 + 3u_3v_3$$

Is this an inner product space? **(6mks)**

(b) Given a matrix A = $\begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$, evaluate e^{At}, where t is an arbitrary parameter. **(7mks)**

(c) If $u = (2 - 5i, 3, -2 - 4i)$ and $v = (6i, 1 + i, 4 + 2i)$ find $2u + v, -2iv$ and if $w = (4 - i,\ 2i,\ 3 + 2i)$ then find $||w||$ **(4mks)**

(d) (i) What is a symmetric matrix?

(ii) Given a symmetric matrix A = $\begin{bmatrix} 7 & 3 \\ 3 & -1 \end{bmatrix}$. Find an orthogonal matrix P which diagonalizes A and hence find $P^t\ AP$. **(8mks)**

(e) (i) Define a bilinear form f on a vector space V

(ii) Show whether if given that f is a bilinear form on V then - f is also a bilinear form on V, where $-f(x, y) = (-f)(x, y)$ given that $-f(x, y) = (ax, +bx_2, y)$

(5mks)

QUESTION TWO (20MKS)

(a) Given a matrix A = $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ -4 & 4 & 3 \end{bmatrix}$. Express A^4 and A^{-2} in terms of A^2, A and I_3 is the 3 x 3 identify matrix. **(10mks)**

(b) Given a matrix A = $\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$. Use Cayley Hamilton's theorem to find the powers of A up to the fifth power. Hence write down the series e^{At}. **(10mks)**

QUESTION THREE (20MKS)

Given a 3 x 3 matrix A = $\begin{bmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \\ -1 & 1 & 2 \end{bmatrix}$.

(a) Find the characteristic polynomial and the eigenvalues of A. **(3mks)**

(b) Determine the eigenvectors corresponding to each eigenvectors λ. **(5mks)**

(c) Find the A.M and G.M of each λ. **(4mks)**

(d) (i) State Cayley – Hamilton's theorem. **(4mks)**

(ii) Is theorem in (d) true for matrix A? **(5mks)**

(e) Diagonalize matrix A. **(3mks)**

QUESTION FOUR (20MKS)

(a) What is a positive matrix? **(2mks)**

(b) Is the given 3 x 3 matrix A = $\begin{bmatrix} -4 & 1+i & 5-2i \\ 1-i & 3 & 7i \\ 5+2i & -7i & -1 \end{bmatrix}$ positive? **(5mks)**

(c) (i) Given that A = $\begin{bmatrix} 1 & 2 & -3 \\ 2 & 5 & -4 \\ -3 & -4 & 8 \end{bmatrix}$ is a real symmetric matrix. Find a non- singular matrix

P such that P^t AP is diagonal. **(7mks)**

(ii) Hence, or otherwise find the signature of A. **(2mks)**

(d) Let f be any symmetric bilinear form on V(F) and let q be the corresponding quadratic form associated with f. Derive the polarization identity.

$(1+1)f(x,y) = q(x+y) - q(x) - q(y)$, with $1 + 1 \neq 0$ **(4mks)**

QUESTION FIVE (20MKS)

(a) If u and v are orthogonal vectors in an inner product space, then state and prove the generalized Pythagoras theorem. **(4mks)**

(b) Let R^4 have an Eudidean inner product. Use Gram-Schmidt process to transform the following basis S into an orthonormal basis,

$S = \{u_1 = (0,2,1,0),\ u_2 = (1,-1,0,0) = u_3 = (1,2,0,-1), u_4 = (1,0,0,1)\}$. **(16mks)**