KABARAK UNIVERSITY

UNIVERSITY EXAMINATIONS

2009/20010 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF SCIENCE IN ECONOMICS AND MATHEMATICS

COURSE CODE: **MATH 220**

COURSE TITLE: **LINEAR ALGEBRA II**

STREAM: **SESSION IV**

DAY: **WEDNESDAY**

TIME: **9.00 – 11.00 A.M.**

DATE: **11/08/2010**

INSTRUCTIONS:

- Attempt question **ONE** and any other **TWO** Questions

PLEASE TURNOVER

QUESTION ONE (30 MARKS) COMPULSORY

(a) (i) Given that $A = \begin{bmatrix} 3 & 2 \\ 5 & 4 \end{bmatrix}$ x = $\begin{pmatrix} x \\ y \end{pmatrix}$ and $C = \begin{pmatrix} 1 \\ -4 \end{pmatrix}$ find A^{-1}, hence solve the matrix equation $Ax = c$ (3 mks)

(ii) Find the value of λ if $A = \begin{bmatrix} 3 & 2\lambda & 4 \\ \lambda & 5 & 3 \\ -1 & 8\lambda & 2 \end{bmatrix}$ is a singular matrix (3 mks)

(iii) Let $A = \begin{bmatrix} 2 & 5 \\ -3 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & -5 \\ 3 & k \end{bmatrix}$ what values of k if any will make AB = BA (3 mks)

(b) (i) Define the terms Eigenvector and Eigenvalue of a matrix A (2 mks)

(ii) Find the Eigenvalues of matrix $A = \begin{bmatrix} 3 & 2 \\ 3 & 8 \end{bmatrix}$ (3 mks)

(c) (i) Let $V = \{v_1,\ v_2,\ v_3 \ldots, v_n\}$ be vector in $\Re^n$ state clearly what is meant by, V is linearly independent. (2 mks)

(ii) Let $V_{1=} \begin{pmatrix} 5 \\ 0 \\ 0 \end{pmatrix},\ V_2 = \begin{pmatrix} 7 \\ 2 \\ -6 \end{pmatrix}, V_3 = \begin{pmatrix} 9 \\ 4 \\ -8 \end{pmatrix}$. Determine if the set $\{v_1\ v_2\ v_3\}$ is linearly independent. (5 mks)

(iii) Show that $\{u_1,\ u_2,\ u_3\}$ are an orthogonal basis for $\Re^3$ respectively

$$u_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, u_2 = \begin{pmatrix} -1 \\ 4 \\ 1 \end{pmatrix}, u_3 = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$$ (3 mks)

(d) (i) Let W be the set of all vectors of the form $\begin{bmatrix} 5b + 2c \\ b \\ c \end{bmatrix}$ where b and c are arbitrary. Find vectors $u\ and\ v$ such that W = span $\{u, v\}$. Hence show that W is a Subspace of $\Re^3$ (4 mks)

(ii) Find a matrix A such that W = Col A given

$$W = \left\{ \begin{bmatrix} 6a - b \\ a + b \\ -7a \end{bmatrix} : a, b\ in\ \Re \right\}$$ (2 mks)

QUESTION TWO (20 MARKS)

(a) Let $A = \begin{pmatrix} 7 & 2 \\ -4 & 1 \end{pmatrix}$ find a matrix for A^k given that $A = PDP^{-1}$ where $P = \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}$ and

$D = \begin{bmatrix} 5 & 0 \\ 0 & 3 \end{bmatrix}$ (5 mks)

(b) Show that the matrix $A = \begin{pmatrix} 2 & 5 \\ 3 & 4 \end{pmatrix}$ satisfies the characteristics equation. (5 mks)

(c) Diagonalise the matrix $\begin{bmatrix} 2 & 2 & -1 \\ 1 & 3 & -1 \\ -1 & -2 & 2 \end{bmatrix}$ if possible. (10 mks)

QUESTION THREE (20 MARKS)

(a) Determine whether $v_1 = (1,1,2), v_2 = (1\ \ 0\ \ 1)$ and $v_3 = (2,1,3)$ span the vector space $\Re^3$ (5 mks)

(b) Let $T: \Re^3 \rightarrow \Re^3$ be the linear mapping defined by;

$T(x,y,z) = (x + 2y - z); y + z, x + y - 2z.$ Find a basis and the dimension of the image T (5 mks)

(c) Determine whether the vectors $v_1 = (1,2,2,1)$ $v_2 = (2,3,4,1)$ and $v_3 = (3,8,7,5)$ in $\Re^4$ are linearly dependent (10 mks)

QUESTION FOUR (20 MARKS)

(a) Define a linear Transformation $T: \Re^2 \rightarrow \Re^2$ by $T(x) = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -x_2 \\ x_1 \end{bmatrix}$

Find the images under T if $u = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$, $v = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $u + v = \begin{pmatrix} 6 \\ 4 \end{pmatrix}$, hence describe the transformation under T (4 mks)

(b) Obtain the Eigenvector of the matrix $A = \begin{bmatrix} 4 & -3 \\ 2 & -1 \end{bmatrix}$ hence find $\begin{pmatrix} 4 & -3 \\ 2 & -1 \end{pmatrix}^8$ (10 mks)

(c) Show that $\{v_1, v_2, v_3\}$ is an orthonormal basis of $\Re^3$ where

$$V_1 = \begin{bmatrix} \frac{3}{\sqrt{11}} \\ \frac{1}{\sqrt{11}} \\ \frac{1}{\sqrt{11}} \end{bmatrix} \quad V_2 = \begin{bmatrix} \frac{-1}{\sqrt{6}} \\ \frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \end{bmatrix} \quad V_3 = \begin{bmatrix} -\frac{1}{\sqrt{66}} \\ -\frac{-4}{\sqrt{66}} \\ \frac{7}{\sqrt{66}} \end{bmatrix}$$

(6 mks)

QUESTION FIVE (20 MARKS)

(a) (i) Define a linear Transformation by $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 1 \\ 3 & 1 & -2 \end{bmatrix}$ (5 mks)

(ii) Write the following equations in matrix form and using the results in (i) above, find x, y and z

$$x + 2y + 3z = 6$$
$$2x + y + z = 5$$
$$3x + y - 2z = 1$$

(5 mks)

(b) (i) Show that the linear system below has no solution

$$x_1 + 2x_2 + x_3 - 2x_4 = 1$$
$$2x_1 + x_2 - x_3 + x_4 = 0$$
$$x_1 - x_2 - 2x_3 + 3x_4 = 1$$

(5 mks)

(ii) Write the matrix $A = \begin{pmatrix} 3 & -1 \\ 1 & -2 \end{pmatrix}$ as a linear combination of $B = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$

C = $\begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix}$ and D = $\begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$ (5 mks)