KABARAK UNIVERSITY

EXAMINATIONS

2008/2009 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF SCIENCE IN ECONOMICS AND MATHEMATICS

COURSE CODE: **MATH 220**

COURSE TITLE: **LINEAR ALGEBRA II**

STREAM: **Y2S2**

DAY: **MONDAY**

TIME: **9.00 -11.00 A.M.**

DATE: **23/03/2009**

INSTRUCTIONS:

(i) Attempt **QUESTION ONE** and **ANY OTHER TWO** questions.
(ii) Show all your working and be neat

PLEASE TURN OVER

QUESTION ONE (30MKS)

(a) If and are any two vectors in an inner product space defined as

$< \; , \; > = 2 \quad + \quad + 3$

Is this an inner product space? **(6mks)**

(b) Given a matrix A = $\begin{matrix} 1 & -1 \\ 2 & -1 \end{matrix}$, evaluate , where t is an arbitrary parameter. **(7mks)**

(c) If $=(2-5 \; , 3, 2-4 \;)$ and $= (6 \; , 1+ \; , 4+2 \;)$ find $2 \; + \; , -2$

and if $= (4- \; , \; 2 \; , \; 3+2 \;)$ then find $\| \; \|$ **(4mks)**

(d) (i) What is a symmetric matrix?

(ii) Given a symmetric matrix A = $\begin{matrix} 7 & 3 \\ 3 & -1 \end{matrix}$. Find an orthogonal matrix P which diagonalizes A and hence find . **(8mks)**

(e) (i) Define a bilinear form on a vector space V

(ii) Show whether if given that is a bilinear form on V then - is also a bilinear form on V, where $- \; (\; , \;) = (- \;)(\; , \;)$ given that $- \; (\; , \;) = (\; , + \; {}_2, \;)$

(5mks)

QUESTION TWO (20MKS)

(a) Given a matrix A = $\begin{matrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ -4 & 4 & 3 \end{matrix}$. Express and in terms of , and is the 3 x 3 identify matrix. **(10mks)**

(b) Given a matrix A = $\begin{matrix} 1 & 2 \\ 3 & 1 \end{matrix}$. Use Cayley Hamilton's theorem to find the powers of A up to the fifth power**.** Hence write down the series . **(10mks)**

QUESTION THREE (20MKS)

Given a 3 x 3 matrix A = $\begin{matrix} 2 & -1 & -1 \\ 1 & 0 & -1 \\ -1 & 1 & 2 \end{matrix}$.

(a) Find the characteristic polynomial and the eigenvalues of A. **(3mks)**

(b) Determine the eigenvectors corresponding to each eigenvectors . **(5mks)**

(c) Find the A.M and G.M of each λ. **(4mks)**

(d) (i) State Cayley – Hamilton's theorem. **(4mks)**

(ii) Is theorem in (d) true for matrix A? **(5mks)**

(e) Diagonalize matrix A. **(3mks)**

QUESTION FOUR (20MKS)

(a) What is a positive matrix? **(2mks)**

(b) Is the given 3 x 3 matrix A = $\begin{matrix} -4 & 1+ & 5-2 \\ 1- & 3 & 7 \\ 5+2 & -7 & -1 \end{matrix}$ positive? **(5mks)**

(c) (i) Given that A = $\begin{matrix} 1 & 2 & -3 \\ 2 & 5 & -4 \\ -3 & -4 & 8 \end{matrix}$ is a real symmetric matrix. Find a non- singular matrix

P such that AP is diagonal. **(7mks)**

(ii) Hence, or otherwise find the signature of A. **(2mks)**

(d) Let f be any symmetric bilinear form on V(F) and let q be the corresponding quadratic form associated with f. Derive the polarization identity.

$(1+1)f(\ ,\ ) = (\ \ +\ ) - (\ ) - (\ )$, with $1 + 1 \neq 0$ **(4mks)**

QUESTION FIVE (20MKS)

(a) If and are orthogonal vectors in an inner product space, then state and prove the generalized Pythagoras theorem. **(4mks)**

(b) Let have an Eudidean inner product. Use Gram-Schmidt process to transform the following basis into an orthonormal basis,

$=\{\ = (0,2,1,0),\ \ = (1,-1,0,0) = \ = (1,2,0,-1),\ \ = (1,0,0,1)\}$. **(16mks)**