KABARAK UNIVERSITY

SUPPLEMENTARY/SPECIAL EXAMINATIONS

2008/2009 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE

COURSE CODE: MATH 220

COURSE TITLE: LINEAR ALGEBRA II

STREAM: **SESSION II**

DAY: **WEDNESDAY**

TIME: **2.00 – 4.00 P.M.**

DATE: **18/03/2009**

INSTRUCTIONS:

1. Answer Question **ONE** and any other **TWO** Questions
2. Show **ALL** your workings.

PLEASE TURN OVER

QUESTION ONE (30 MARKS)

(a) Define the terms eigenvector and eigenvalue of a matrix A (2 mks)

(b) Find the eigenvalues of the matrix

$$A = \begin{bmatrix} 3 & 2 \\ 3 & -2 \end{bmatrix}$$ (4 mks)

(c) Show that the matrix

$$A= \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ -3 & 5 & 2 \end{bmatrix}$$

Is nondiagonalizable (8 mks)

(d) Show that the matrix

$$A= \begin{bmatrix} 2 & 5 \\ 3 & 4 \end{bmatrix}$$

Satisfies its characteristics equation (5 mks)

(e) If V is an inner product space ad ***x*** and ***y*** are vectors in V, prove that

$\|x+y\|^2 = \|x\|^2 + \|y\|^2$ iff (x,y) = 0 (4 mks)

(f) Prove that an orthogonal set of nonzero vectors in inner product space is linearly independent (7 mks)

QUESTION TWO (20 MARKS)

(a) Let A be an nxn orthogonal matrix and let ***x*** and ***y*** be nonzero vectors in R^n.

Prove that

(i) $|x| = |Ax|$ (3 mks)

(ii) The angle between ***Ax*** and ***Ay*** is equal to the angle between x and y. (4 mks)

(b) Consider the matrix

$$A = \begin{bmatrix} \frac{2}{3} & \frac{2}{3} & \frac{-1}{3} \\ \frac{2}{3} & \frac{-1}{3} & \frac{2}{3} \\ \frac{-1}{3} & \frac{2}{3} & \frac{2}{3} \end{bmatrix}$$

(i) Show that A is an orthogonal matrix (3 mks)

(ii) Let f be the linear transformation from R^3 given by f(x) = Ax. Write down the images under f of the standard basis vectors. (4 mks)

(iii) Evaluate Ax and Ay where

$$x = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad \text{and} \quad y = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$$

and verify that $|x| = |Ax|$ and $|y| = |Ay|$ (6 mks)

QUESTION THREE (20 MKS)

(a) Find an orthonormal basis for

$$W = \{(x, y, z) \ | x + y + z = 0 \text{ in } R^3 \}$$

With respect to the dot product. (10 mks)

(b) Find the matrix of the orthogonal projection onto the subspace of R^4 spanned by

$$V_1 = \frac{1}{2}\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \quad V_2 = \frac{1}{2}\begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}$$

(4 mks)

(c) Show that the set

$$V = \{(0,1,0), \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right), \left(\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}\right)\}$$

Is orthonormal (6 mks)

QUESTION FOUR (20 MKS)

(a) Find a diagonal matrix D similar to the matrix

$$A=\begin{bmatrix} 1 & 0 & 0 \\ -8 & 4 & -6 \\ 8 & 1 & 9 \end{bmatrix}$$ (14 mks)

(b) (i) For the matrix A above (i) find A^n if n is a positive integer (2 mks)

(ii) Find A^3 (2 mks)

(iii) Find $A^{½}$ (2 mks)

QUESTION FIVE (20 MARKS)

Consider the matrix

$$A=\begin{bmatrix} 3 & 3 & 2 \\ 2 & 4 & 2 \\ -1 & -3 & 0 \end{bmatrix}$$

(i) Find the eigenvalues of A (5 mks)

(ii) Find the algebraic and geometric multiplicities of each eigenvalue (9 mks)

(iii) Find a basis for each eigenspace. (6 mks)