KABARAK UNIVERSITY

UNIVERSITY EXAMINATIONS

2008/2009 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE

COURSE CODE: MATH 220

COURSE TITLE: LINEAR ALGEBRA II

STREAM: SESSION IV

DAY: MONDAY

TIME: 2.00 – 4.00 P.M.

DATE: 30/11/2009

INSTRUCTIONS:

Attempt question **ONE** and any other **TWO** questions

PLEASE TURN OVER

1. Consider the matrix $B \begin{pmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 1 & 2 & 3 \end{pmatrix}$

(a) Find for the matrix B,

(i) The characteristics equation **(2 marks)**

(ii) The eigenvalues $(D_1, \ D_2, \ D_3 \ )$ **(1 mark)**

(b) If $A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 0 \\ 1 & 2 & 1 \end{pmatrix}$ Determine A^T, $\det A$ and A^{-1} **(3 marks)**

(8 marks)

(c) Define Orthogonal matrices **(2 marks)**

(d) Determine x and y if

(i) $\begin{pmatrix} x+y & 2 \\ 1 & x-y \end{pmatrix} = \begin{pmatrix} 3 & 2 \\ 1 & 5 \end{pmatrix}$ **(2 marks)**

(ii) Solve $\begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$ $(a \ \ b \ \ c \ \ d)$ **(1 mark)**

(e) Det AB = Det A Det B where A and B are some square matrices. Verify for $A_{2 \times 2}$ and $B_{2 \times 2}$ **(10 marks)**

(f) (i) Determine the eigenvalues and eigenvectors of $q(x, \ y) = \ 5x^2 - \ 4xy + \ 5y^2$

(9 marks)

(ii) Classify the above quadratic form **(1 mark)**

2. (a) Use Cramers rule to solve the system of equations

$$2x + y - 3z = \ -3$$
$$x - y + 4z = 8$$
$$3x + 2y + z = 7$$

(8 marks)

(b) Solve the system

$$x_1 + x_2 - x_4 = 3$$
$$-x_1 + 2x_2 + x_3 = -4$$
$$-x_2 + x_3 + 2x_4 = 1$$
$$3x_2 - 3x_3 + x_4 = -3$$

(10 marks)

(c) Given the quadratic form $q(x) = q(r,s) = r^2 + 4rs - 2s^2$ find the eigen values.

(2 marks)

3. (a) Classify the following quadratic form as positive, definite, positive semi-definite, negative definite, negative semi-definite or indefinite.

(i) $q(\underline{x}) = 2x^2 + 6xy + 2y^2$ **(4 marks)**

(ii) $\text{q}(\underline{x}) = x^2 + 4xy - 2y^2$ **(4 marks)**

(b) Classify; $q(x,\ y,\ z) = x^2 + y^2 + z^2\ 4xy + 4xz + 4yz$ **(7 marks)**

(c) Given matrix $A = \begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix}$ and matrix $B = \begin{pmatrix} 7 & 2 \\ -3 & 4 \end{pmatrix}$ prove that det AB = det A det B **(5 marks)**

4. (a) Use cramers rule to solve the system of equations.

$$x_1 - x_2 + x_3 = 3$$
$$2x_1 + x_2 + 2x_2 = 3$$
$$3x_1 + 2x_2 - x_3 = 0$$

(9 marks)

(b) Compute the inverse of the matrix

$$\begin{bmatrix} 2 & -1 & 3 \\ 1 & 0 & -1 \\ 0 & -2 & 1 \end{bmatrix}$$

(7 marks)

(c) Classify the following quadratic form $q(x,y) = 3x^2 - 2xy + 3y^2$ **(4 marks)**

5. (a) Let f be a mapping

$$f: \Re^2 \rightarrow \Re \text{ such that } f\left\{\begin{pmatrix}1\\1\end{pmatrix}\right\} = \begin{pmatrix}3\\2\end{pmatrix}, \ f\left\{\begin{pmatrix}1\\-1\end{pmatrix}\right\} = \begin{pmatrix}2\\1\end{pmatrix}$$

Determine

(i) The standard transformation matrix for f **(5 marks)**

(ii) What will $f\left\{\begin{pmatrix}2\\2\end{pmatrix}\right\}$ be mapped onto **(2 marks)**

(b) (i) Obtain the eigenvectors of the matrix $A = \begin{pmatrix}2 & 1\\1 & 2\end{pmatrix}$ **(6 marks)**

(ii) Hence find $\begin{pmatrix}2 & 1\\1 & 2\end{pmatrix}^5$ **(4 marks)**