**KABARAK UNIVERSITY**

# UNIVERSITY EXAMINATIONS

## 2010/2011 ACADEMIC YEAR

## FOR THE DEGREE OF BACHELOR OF SCIENCE IN ECONOMICS AND MATHEMATICS

**COURSE CODE:** **MATH 220**

**COURSE TITLE:** **LINEAR ALGEBRA II**

**STREAM:** **Y2S2**

**DAY:** **WEDNESDAY**

**TIME:** **9.00 – 11.00 A.M.**

**DATE:** **16/03/2011**

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**INSTRUCTIONS:**

1. Answer question **ONE** and any other **TWO** questions
2. Begin each question on a separate page
3. Show your workings clearly

**PLEASE TURN OVER**

## QUESTION ONE (30 MARKS)

a) Determine if the following sets of vectors are linearly independent or linearly dependent.

i) $P_1 = 1$, $p_2 = x$ and $p_3 = x^2$ in $p^2$ (3 marks)

ii) $P_1 = x – 3$ , $p_2 = x^2 + 2x$ and $p_3 = x^2 + 1$ in $p_2$ (4 marks)

iii) $P_1 = 2x^2 – x + 7$ , $p_2 = x^2 + 4x + 2$ and $p_3 = x^2 – 2x + 4$ in $p_2$ (4 marks)

b) Determine if each of the sets of vectors will be a basis for $\Re^3$ .

(i) $v_1$ □$\tilde{1}$,1,1) , $v_2$ □0,1, 2) and $v_3$□3,$\tilde{0}$,1) (5 marks)

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(ii) $v_1$□1,0,0) ,$v_2$ □0,1,0) and $v_3$ □0,0,1) (5 marks)

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(iii) $v_1$□1,1,0) and $v_2$$\tilde{□}$1,0,0) (4 marks)

(iv) $v_1$ □$\tilde{1}$,1,1) , $v_2$ $\tilde{□}$1, $\tilde{2}$,2) and $v_3$ $\tilde{□}$1, $\tilde{4}$,4) (5 marks)

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## QUESTION TWO (20) MARKS)

a) Determine if the given set is a subspace of the given vector space.

(i) Let *W* be the set of all points, *x*, *y*, from $\Re^2$ in which *x* □0. Is this a subspace of $\Re^2$ ?
(2 marks)

(ii) Let *W* be the set of all points from $\Re^3$ of the form $(0, x_2, x_3)$. Is this a subspace of $\Re^3$ ?
(3 marks)

(iii) Let *W* be the set of all points from $\Re^3$ of the form $(1, x_2, x_3)$ . Is this a subspace of $\Re^3$ ?
(3 marks)

b) Determine if the following sets of vectors will span $\Re^3$

(i) $v_1$ □2,0,1) , $v_2$ $\tilde{□}$1,3, 4 ), and $v_3$ □1,$\tilde{1}$,2) (6 marks)

(ii) $v_1$□1,$\tilde{2}$,1) , $v_2$ □$\tilde{3}$,1,1) , and $v_3$ $\tilde{□}$3,$\tilde{8}$,5) (6 marks)

## QUESTION THREE ( 20 MARKS)

a) Reduce each of the following sets of vectors to obtain a basis for the given vector spaces

i) $v_1 = (1,0,0)$ , $v_2 = (0,1,-1)$ , $v_3 = (0,4,-3)$ and $v_4 = (0,2,0)$ for $\Re^3$ (4 marks)

ii) $p_0= 2$ , $p1 = -4x$ , $p_2 = x^2 + x + 1$ , $p_3 = 2x + 7$ and $p_4 = 5x^2 -1$ for $p_2$ (4 marks)

b) For each of the following compute $\langle u, v\rangle$, $\|u\|$ and $d(\mathbf{u}, \mathbf{v})$ for the given pair of vectors and inner product.

(i) $\mathbf{u} = (2, -1, 4)$ and $\mathbf{v} = (3, 2, 0)$ in $\Re^3$ with the standard Euclidean inner product. (4 marks)

(ii) $\mathbf{u} = (2, -1, 4)$ and $\mathbf{v} = (3, 2, 0)$ in $\Re^3$ with the weighed Euclidean inner product using the weights $w_1 = 2$, $w_2 = 6$ and $w_3 = 1/5$. (4 marks)

(iii) $\mathbf{u} = x$ and $\mathbf{v} = x^2$ in $C[0,1]$ using the inner product for continous functions. (4 marks)

## QUESTION FOUR (20 MARKS)

a) Given that $v_1 = (2,-1,0)$, $v_2 = (1,0,-1)$, and $v_3 = (3,7,-1)$ is a basis of $\Re^3$ and assuming that we're working with the standard Euclidean inner product construct an orthogonal basis for $\Re^3$. (11 marks)

b) Determine if the given set is a subspace of the given vector space.

(i) Let $W$ be the set of diagonal matrices of size $n \times n$. Is this a subspace of $M_{22}$? (3 marks)

(ii) Let $W$ be the set of matrices of the form $\begin{bmatrix} 0 & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$. Is this a subspace of $M_{32}$? (3 marks)

(iii) Let $W$ be the set of matrices of the form $\begin{bmatrix} 2 & a_{12} \\ 0 & a_{22} \end{bmatrix}$. Is this a subspace of $M_{22}$? (3 marks)

## QUESTION FIVE (20 MARKS)

Find all the Eigen values and Eigen vectors for the given matrices

i) $A = \begin{bmatrix} 4 & 0 & 1 \\ -1 & -6 & -2 \\ 5 & 0 & 0 \end{bmatrix}$ (10 marks)

ii) $A = \begin{bmatrix} 6 & 3 & -8 \\ 0 & -2 & 0 \\ 1 & 0 & -3 \end{bmatrix}$ (10 marks)