KABARAK UNIVERSITY

UNIVERSITY EXAMINATIONS

2008/2009 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE

COURSE CODE: MATH 220

COURSE TITLE: LINEAR ALGEBRA II

STREAM: SESSION **IV**

DAY: **WEDNESDAY**

TIME: **9.00 – 11.00 A.M.**

DATE: **26/11/2008**

INSTRUCTIONS TO CANDIDATES:

1. Answer Question **ONE** and any other **TWO** Questions

PLEASE TURN OVER

QUESTION ONE (30 MARKS)

(a) If V is an inner product space and x and y are vectors in V, prove that

$$\|x+y\|^2 = \|x\|^2 + \|y\|^2, \text{ iff } (x, y) = 0$$ **(3 mks)**

(b) (i) Define the terms eigenvalue and eigenvector in relation to a matrix A. **(2 mks)**

(ii) Determine the eigenvalues of the matrix

$$A = \begin{bmatrix} 2 & 5 \\ 3 & 4 \end{bmatrix}$$ **(4 mks)**

(c) Determine the determinant of the matrix

$$A = \begin{bmatrix} 2 & 3 & 4 & 6 \\ 2 & 0 & -9 & 6 \\ 4 & 1 & 0 & 2 \\ 0 & 1 & -1 & 0 \end{bmatrix}$$

by cofactor method **(9 mks)**

(d) Show that the set

$$V = \left\{(0,1,0)\left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}\right)\right\}$$ is orthonormal. **(6 mks)**

(e) Diagonalize the matrix $\begin{bmatrix} p & -q \\ q & p \end{bmatrix}$ where p and q are real numbers and $q \neq 0$. **(6 mks)**

QUESTION TWO (20 MARKS)

Consider the matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 0 & 2 & 1 \\ 4 & -4 & -1 \end{bmatrix}$

(a) Find the eigenvalues of A. **(5 mks)**

(b) Find the algebraic and geometric multiplicities of each eigenvalue. **(9 mks)**

(c) Find a basis for each eigenspace **(6 mks)**

QUESTION THREE (20 MKS)

(a) (i) Show that the determinant of a diagonal matrix is the product of its main diagonal entries. **(3 mks)**

(ii) Show that the eigenvalues of an upper or lower triangular matrix are the entries on the main diagonal. **(3 mks)**

(b) Diagonalize the real symmetric matrix

$$\begin{bmatrix} 2 & 2 & 0 \\ 2 & 5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$ through orthogonal reduction. **(14 mks)**

QUESTION FOUR (20 MKS)

(a) (i) State the Cayley - Hamilton theorem. **(2 mks)**

(ii) Verify the Cayley – Hamilton theorem for the matrix

$$A = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix}$$ and hence determine A^{-1} and A^4. **(8 mks)**

(b) Find an orthonormal basis for W = $\{(x, y, z) : x + y + z = 0, in \Re^3\}$ with respect to the dot product. **(10 mks)**

QUESTION FIVE (20 MARKS)

(a) (i) If V is an inner product space and x and y are vectors in V, prove that

$$\| x+y \| \leq \| x \| + \| y \|$$ **(4 mks)**

(ii) Apply the Gram- Schruidt process to V = $\{V_1, = (0,-1,1), V_2 = (-1,0,1)\}$ **(9 mks)**

(b) Let A be an n x n orthogonal matrix and let x and y be non- Zero vectors in $\Re$. Show that;

(i) $|x| = |Ax|$ **(3 mks)**

(ii) The angle between Ax and Ay is equal to the angle between x and y. **(4 mks)**