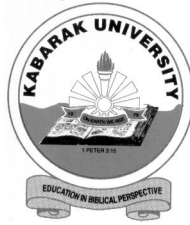


**KABARAK**



**UNIVERSITY**

**UNIVERSITY EXAMINATIONS**

**2008/2009 ACADEMIC YEAR**

**FOR THE DEGREE OF BACHELOR OF EDUCATION  
SCIENCE**

**COURSE CODE:** MATH 220

**COURSE TITLE:** LINEAR ALGEBRA II

**STREAM:** SESSION IV

**DAY:** WEDNESDAY

**TIME:** 9.00 – 11.00 A.M.

**DATE:** 26/11/2008

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**INSTRUCTIONS TO CANDIDATES:**

1. Answer Question **ONE** and any other **TWO** Questions

**PLEASE TURN OVER**

### QUESTION ONE (30 MARKS)

(a) If  $V$  is an inner product space and  $x$  and  $y$  are vectors in  $V$ , prove that

$$\|x + y\|^2 = \|x\|^2 + \|y\|^2, \text{ iff } (x, y) = 0 \quad (3 \text{ mks})$$

(b) (i) Define the terms eigenvalue and eigenvector in relation to a matrix  $A$ .

(2 mks)

(ii) Determine the eigenvalues of the matrix

$$A = \begin{bmatrix} 2 & 5 \\ 3 & 4 \end{bmatrix} \quad (4 \text{ mks})$$

(c) Determine the determinant of the matrix

$$A = \begin{bmatrix} 2 & 3 & 4 & 6 \\ 2 & 0 & -9 & 6 \\ 4 & 1 & 0 & 2 \\ 0 & 1 & -1 & 0 \end{bmatrix}$$

by cofactor method

(9 mks)

(d) Show that the set

$$V = \left\{ (0, 1, 0), \left( \frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} \right), \left( \frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}} \right) \right\} \text{ is orthonormal.} \quad (6 \text{ mks})$$

(e) Diagonalize the matrix  $\begin{bmatrix} p & -q \\ q & p \end{bmatrix}$  where  $p$  and  $q$  are real numbers and  $q \neq 0$ . (6 mks)

### QUESTION TWO (20 MARKS)

Consider the matrix  $A = \begin{bmatrix} 1 & 1 & 3 \\ 0 & 2 & 1 \\ 4 & -4 & -1 \end{bmatrix}$

(a) Find the eigenvalues of  $A$ .

(5 mks)

(b) Find the algebraic and geometric multiplicities of each eigenvalue. **(9 mks)**

(c) Find a basis for each eigenspace **(6 mks)**

**QUESTION THREE (20 MKS)**

(a) (i) Show that the determinant of a diagonal matrix is the product of its main diagonal entries. **(3 mks)**

(ii) Show that the eigenvalues of an upper or lower triangular matrix are the entries on the main diagonal. **(3 mks)**

(b) Diagonalize the real symmetric matrix

$$\begin{bmatrix} 2 & 2 & 0 \\ 2 & 5 & 0 \\ 0 & 0 & 3 \end{bmatrix} \text{ through orthogonal reduction.} \quad \mathbf{(14 mks)}$$

**QUESTION FOUR (20 MKS)**

(a) (i) State the Cayley - Hamilton theorem. **(2 mks)**

(ii) Verify the Cayley – Hamilton theorem for the matrix

$$A = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix} \text{ and hence determine } A^{-1} \text{ and } A^4. \quad \mathbf{(8 mks)}$$

(b) Find an orthonormal basis for  $W = \{(x, y, z) : x + y + z = 0, \text{ in } \mathfrak{R}^3\}$  with respect to the dot product. **(10 mks)**

**QUESTION FIVE (20 MARKS)**

(a) (i) If  $V$  is an inner product space and  $x$  and  $y$  are vectors in  $V$ , prove that

$$\|x + y\| \leq \|x\| + \|y\| \quad (4 \text{ mks})$$

(ii) Apply the Gram- Schruidt process to  $V = \{V_1, = (0, -1, 1), V_2 = (-1, 0, 1)\}$  (9 mks)

(b) Let  $A$  be an  $n \times n$  orthogonal matrix and let  $x$  and  $y$  be non- Zero vectors in  $\mathfrak{R}$  . Show that;

(i)  $|x| = |Ax|$  (3 mks)

(ii) The angle between  $Ax$  and  $Ay$  is equal to the angle between  $x$  and  $y$ . (4 mks)