KABARAK UNIVERSITY

UNIVERSITY EXAMINATIONS

2010/2011 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF SCIENCE IN ECONOMICS AND MATHEMATICS

COURSE CODE: MATH 220

COURSE TITLE: LINEAR ALGEBRA II

STREAM: SESSION IV

DAY: FRIDAY

TIME: 2.00 – 4.00 P.M.

DATE: 26/11/2010

INSTRUCTIONS:

- Attempt question **ONE** and any other **TWO** Questions

PLEASE TURNOVER

QUESTION ONE (30 MARKS)

a) Let $V = \{v_1, v_2, v_3, ..., v_n\}$ be a vector in R^n. Explain clearly what is meant by saying V is linearly independent.

b) Let V be a vector space in R^3 and let $v_1 = (1,2,1)$, $v_2 = (1,0,2)$ and v_3=$(1,1,0)$. Does v_1, v_2, and v_3 span V?

c) Show that the following matrix is diagonalizable and find a diagonal matrix similar to the given matrix $\begin{pmatrix} 2 & 0 & 0 \\ 1 & -1 & -2 \\ -1 & 0 & 1 \end{pmatrix}$

d) Compute the eigenvalues and hence the eigenvectors of the matrix $\begin{pmatrix} 2 & 0 & 0 \\ 3 & -1 & 0 \\ 0 & 4 & 3 \end{pmatrix}$.

e) Let $T : R^2 \to R^3$ be a linear transformation such that $T(1) = 1$, $T(x) = x^2$ and $T(x^2) = x^3 + x$, determine;

$$T(2x^2 - 5x + 3)$$
$$T(ax^2 + bx + c)$$

f) On □$[a,b]$ for $b > a$, define $\langle f, g \rangle = \int_a^b f(x)g(x)dx$. Verify that

$\langle f, g \rangle = \int_a^b f(x)g(x)dx$ is an inner product.

QUESTION TWO (20 MARKS)

a) Solve the following system of equations using row reduction method

$$x + 3y + 6z = 25$$
$$2x + 7y + 14z = 58$$
$$2y + 5z = 19$$

b) Let V be an inner product space and let x and y be vectors in V. Prove that $\|(x, y)\| \leq \|x\|\|y\|$

c) Show that $\begin{pmatrix} 2 & 5 \\ 5 & 2 \end{pmatrix}$ and $\begin{pmatrix} 0 & -3 \\ 3 & 0 \end{pmatrix}$ are orthogonal with respect to the inner product defined by $\langle A, B \rangle = a_{11}b_{11} + a_{12}b_{12} + a_{21}b_{21} + a_{22}b_{22}$ for $A, B \in M_{2x2}$

QUESTION THREE (20 MARKS)

a) Show that $B = \left\{\left(\frac{2}{3}, \frac{1}{2}, \frac{2}{3}\right), \left(\frac{-2}{3}, \frac{2}{3}, \frac{1}{3}\right)\right\}$ is an orthogonal basis for the inner product space $W = \{(x, y, z) \in R^3 : x + 2y - 2z = 0\}$ with respect to the dot product.

b) Find the orthogonal basis for the space $\{(3t,-4t,12t):T\in R\}$ or R^3 with respect to the dot product.

c) Find the norm of $\begin{pmatrix} -4 & 0 \\ 1 & 8 \end{pmatrix}$, in the inner product defined by $\langle A,B\rangle = a_{11}b_{11}+a_{12}b_{12}+a_{21}b_{21}+a_{22}b_{22}$.

d) Verify the Cauchy-Schwartz inequality for the vectors $x_1=(-4,3,0,12)$ and $x_2=(-2,0,1,2)$ in R^4 with respect to the dot product.

QUESTION FOUR (20 MARKS)

a) What condition must real symmetric matrix have for it to be diagonalizable?

b) Which of the following matrices are diagonalizable?

i). $\begin{pmatrix} 2 & -1 \\ 1 & 0 \end{pmatrix}$ ii). $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$, iii). $\begin{pmatrix} 3 & 4 \\ -1 & 2 \end{pmatrix}$

c) Find an invertible matrix P for the following matrix $A=\begin{pmatrix} 2 & 2 & -2 \\ 1 & 3 & -1 \\ -1 & 1 & 1 \end{pmatrix}$

QUESTION FIVE (20 MARKS)

a) Show that the function T with domain in the set of 2x1 vectors and defined by $T\left[\begin{pmatrix} x \\ y \end{pmatrix}\right]=x$ is linear.

b) Solve the following system of equations using row reduction method.

$$x_1+2x_2-3x_3=0$$
$$2x_1-2x_2-x_3=-1$$
$$-3x_1+5x_2+x_3=3$$

c) Find the $Ker\ T$ for the following linear transformation: $T:R^3\to R^3$ defined by $T(x,y,z)=:\{(x+y+z),(3x-y-z),(5x+y-3z)\}$.

d) Consider the set defined by $V=\{x,y,z:ax+by+cz=0,\quad a,b,c\in R\}$, check if V is a vector space.

QUESTION SIX (20 MARKS)

1. Let $\dim_R V = 2$ and define T on V by

$$v_1T = av_1 + bv_2$$
$$v_2T = xv_1 + yv_2,$$

where $a,b,x,y \in R$. In terms of a,b,x,y, find necessary and sufficient conditions that T have two distinct eigenvalues in R.

2. Let V be a vector space over the field F and let D be the operator in V which computes the derivative of the polynomial in x defined by $\left(a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4\right)D = a_1 + 2a_2x + 3a_3x^2 + 4a_4x^3$. Consider the basis $\{v_1 = 1, v_2 = 1+x, v_3 = 1+x^2, v_4 = 1+x^3\}$. Find the matrix m(D) of D in this basis.

3 Let matrix $A = \begin{pmatrix} 3 & 2 & 1 \\ 1 & -1 & 2 \\ 2 & 4 & -1 \end{pmatrix}$ *and* $B = \begin{pmatrix} -7 & 6 & 5 \\ 5 & -8 & -5 \\ 6 & -8 & -5 \end{pmatrix}$.

Find AB hence or otherwise solve the system of equations

$$3x + 2y + z = 9$$
$$x - 2y + 3y = 6$$
$$2x + 4y - z = 5$$

4. $V = \{v_1, v_2, v_3, ..., v_n\} \in R^n$, check whether V is linearly independent.

5. Let $A = \begin{pmatrix} 3 & -2 \\ 4 & -5 \end{pmatrix}$ be a matrix of transformation in the basis $v_1 = (1,0)$ $v_2 = (0,1)$. Find the matrix of m(T) of T in the basis $u_1 = (1,2)$ $u_2 = (2,5)$

6. Let $F : R^2 \to R^2$ be defined by $F(x, y) = (2x + 3y, 4x - 5y)$
Find the T of F relative to the basis $S = (u_1, u_2) = \{(1,-2), (2,-5)\}$

7. Let V be the vector space of polynomials of degree 3 or less over the reals. Define T from V by $(a_0 + a_1v_1 + a_2v_2 + a_3v_3)T = a_0 + a_1(1+x) + a_2(1+x)^2 + a_3(1+x)^3$. Show that T is a linear transformation in V and find the matrix representation in the following basis, $1,\ 1+x,\ 1+x^2,\ 1+x^3$.