KABARAK UNIVERSITY

UNIVERSITY EXAMINATIONS

2010/2011 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE

COURSE CODE: **MATH 220**

COURSE TITLE: **LINEAR ALGEBRA II**

STREAM: **SESSION II**

DAY: **THURSDAY**

TIME: **9.00 – 11.00 A.M.**

DATE: **14/04/2011**

INSTRUCTIONS:

Answer question **ONE** and any other **TWO** questions

PLEASE TURN OVER

QUESTION ONE (3OMARKS) COMPULSORY

(a) Let $T:R^2 \rightarrow R^2$ be defined by $T(x, y) = (x + y, x)$ show that T is linear (3mks)

(b) Given that $= \begin{pmatrix} 1 & -1 & 2 \\ 3 & -3 & 6 \\ -2 & 2 & -4 \end{pmatrix}$

Determine

(i) the rank of A (3mks)

(ii) the null space of A (3mks)

(c) Show that the following defines an inner product inR^2

$<u, v> = x_1 y_1 - x_1 y_2 - x_2 y_1 + 3x_2 y_2$ where $u(x_1 x_2)$ and $v(y_1 y_2)$ (3mks)

(e) show that the vectors (1,1,8,1),(1,0,3,0) and (3,1,14,1) and linearly dependent (3mks)

(f) Solve the following system of equation using row reduction method

$4x_1 - 2x_2 + 3x_3 = 4$

$5x_1 - x_2 + 4x_3 = 7$

$3x_1 + 5x_2 + x_3 = 7$ (7mks)

(g) Determine the eigenvalues and its corresponding eigenvector of

$$\begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$ (6mks)

(h) Let $=\begin{pmatrix}1 & 1\\ -2 & 4\end{pmatrix}$ Show that A is diagonalizable and find a similar matrix to A (3mks)

QUESTION TWO (20MKS)

(a) (i) let $V=(V_1\ V_2\ldots\ldots\ldots V_n)$ be vectors in R^n. Explain clearly what the term linearly dependent means in reference to the vectors in R^n (2mks)

(ii) Show that the set V= {(x, y, z)| 2x –y –z = 0} and K= {(x, y, z) |x + 2y +3z = 0} be subspace of R^3 find H∩K and determine whether it is a subspaceof R^3 (7mks)

QUESTION THREE (20 MKS)

Let U = (1, 3, -4, 2), V = (4, -2, 2, 1) and W= (5, -1, -2, 6) in R^4

(a) Show that <3U - 2V, W> = 3<U, W> -2<V, W> (5mks)

(b) Let f(t) = 3t -5 and g(t) = t^2 be the polynomial space p(t) with inner product

<f, g> =∫ ( ) ( )

Determine

(a) <f, g> (4mks)

(b) (i) ‖f‖ (3mks)

(ii)‖ g ‖ (3mks)

(b) Let c[a; b] denotes the vector space of all continuous functions on the closed interval [a:b]. Let also f(t) and g(t) be functions in c[a:b]. Verify that ∫ ( ) ( ) is an inner product. (5mks)

QUESTION FOUR (20MKS)

(a) Let V_1= (1, 2, 3), V_2 = (-1, 2, 1) and V_3 = (2, 7, 2).Show that (-3, 1, -2) is in the space span by (V_1 $V_2$$V_3$). (5mks)

(b) Let $= \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$

Determine

(i) The null space of A (4mks)

(ii) The general solution to the equation $AX = \begin{matrix} 6 \\ 15 \end{matrix}$ in terms of a particular solution and an arbitrary member of the null space of A (5mks)

(c) Determine the basis for the null space of $\begin{matrix} 1 & 1 & 1 \\ 1 & -1 & 1 \end{matrix}$ (6mks)

QUESTION FIVE (20MKS)

(a) Show that the mapping T: $R^2 \rightarrow R^3$ given by T (X, y) = (X+1, 2Y, X + Y) is not linear. (3mks)

(b) Determine the value of K so that

$$X + 2y - 3z = 4$$
$$3x - y + 5z = 2$$
$$4x + y + (k^2 - 14)z = k+2$$

Has

(i) No solution (3mks)

(ii) Unique solution (3mks)

(iii) Many solution (3mks)

(c) Determine the value of K such that V= (1, 2, k, 3) and V = (3, k, 7, -5) in R^4 are orthogonal. (3mks)

(d) Give A = $\begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{pmatrix}$

Determine the matrix B which is similar to the matrix A. (5mks)