

KABARAK



UNIVERSITY

EXAMINATIONS

2008/2009 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF COMMERCE

COURSE CODE: PHYS 220

**COURSE TITLE: INTRODUCTION TO QUANTUM
MECHANICS.**

STREAM: SESSION V

DAY: FRIDAY

TIME: 11.00A.M -1.00 P.M

DATE: 28/11/2008

INSTRUCTIONS:

Answer question **ONE**, and any other **TWO** questions. Question ONE carries 30 marks, and the others 20 marks each.

PLEASE TURN OVER

CONSTANTS

Speed of light	$c = 3.0 \times 10^8 \text{ m/s}$
Plank's constant	$h = 6.626 \times 10^{-34} \text{ J.s}$ $\hbar = 1.054 \times 10^{-34} \text{ eV.s}$
electron charge	$e = 1.6 \times 10^{-19} \text{ C}$
One electronvolt,	$1 \text{ eV} = 1.6 \cdot 10^{-19} \text{ J}$
Rest mass of an electron	$M_e = 9.1 \times 10^{-31} \text{ kg}$
Rest mass of a neutron	$M_n = 1.675 \times 10^{-27} \text{ kg} = 1.008654 \text{ u}$ $= 939.6 \text{ MeV}/c^2$
Rest mass of a proton	$M_p = 1.672 \times 10^{-27} \text{ kg} = 1.007766 \text{ u}$
One atomic mass unit	$u = 1.66 \times 10^{-27} \text{ kg} = 931 \text{ MeV}/c^2$
Compton formula	$\lambda' - \lambda = \frac{h}{M_o c} (1 - \cos \phi)$
Stefan's constant	$\sigma = 5.670 \times 10^{-8} \text{ w/m}^2 \cdot \text{k}^4$
Atomic mass of $^{238}\text{U}_{92}$	$= 238.050786 \text{ u}$
Atomic mass of $^{234}\text{Th}_{90}$	$= 228.0436 \text{ u}$
Atomic mass of $^{12}\text{C}_6$	$= 12.0000 \text{ u}$
Atomic mass of $^{16}\text{O}_8$	$= 15.994915 \text{ u}$
Atomic mass of $^4\text{He}_2$	$= 4.0026 \text{ u}$
Permittivity of free space ϵ_o	$= 8.854 \times 10^{-12} \text{ F/m}$
Paschen series formula	$\frac{1}{\lambda} = R \left(\frac{1}{3^2} - \frac{1}{n^2} \right)$
Rydberg's constant	$R = 1.097 \times 10^7 \text{ m}^{-1}$
Atomic spacing d	$d = \left[\frac{M}{k\rho} (1.66 \times 10^{-27} \text{ kg} / \text{u}) \right]^{\frac{1}{3}}$
Wien's constant ω_c	$= 2.898 \times 10^{-3} \text{ m.K}$
Permittivity of free space ϵ_o	$= 8.85 \times 10^{-12} \text{ C}^2 / (\text{N.m}^2)$

Question 1 (30 Marks)

- (a) Explain why there was need in the early days to introduce quantum mechanics. (1 mark)
- (b) (i) Define massless particles and give one example. (2 marks)
- (ii) Write down the expressions for the relativistic mass and the relativistic linear momentum of a particle moving at a velocity v . (2 marks)
- (iii) Find the speed of an electron whose total energy is twice its rest mass energy. (2 ½ marks)
- (c) (i) Sketch a well labelled blackbody spectrum of two objects at temperatures T_1 and T_2 , where $T_2 > T_1$. (2 marks)
- (ii) Explain the meaning of the term thermal equilibrium of a body. (1 mark)
- (d) Starting with the expression for the energy (eV) of a single photon, find the voltage that needs to be applied to an x-ray tube for it to emit x-rays with a maximum frequency of 6×10^{18} Hz. (2 marks)
- (e) (i) Sketch the variation of the photoelectron current and the retarding potential of photoelectrons emitted from three different metals whose work function values are related as $\phi_1 < \phi_2 < \phi_3$. (1 ½ marks)
- (ii) A metal surface illuminated by 8.5×10^{14} Hz light emits electrons whose maximum energy is 0.52 eV. The same surface illuminated by 12×10^{14} Hz emits electrons whose maximum energy is 1.97 eV. From the data, find Plank's constant and the work function of the surface. (4 marks)
- (f) (i) With the aid of a clearly labeled diagram, explain briefly how x-rays are produced. (3 marks)
- (ii) Write down the condition for constructive interference of x-rays of wavelength λ scattered at an angle θ from a crystal whose inter atomic spacing is d . (1 mark)
- (iii) Find the distance between adjacent KCl atoms if formula mass of a KCl crystal is 74.55u and its density is 1.98 g/cm^3 . (2 marks)
- (iv) Find the smallest angle of Bragg scattering for 0.3nm x-rays scattered from a crystal of KCl. (2 marks)
- (iv) Define the continuous spectra. (1 mark)
- (g) (i) State the uncertainty principle and write down its expression. (2 mark)

(ii) Define radioactivity. (1 mark)

Question 2 (20 Marks)

(a) (i) Write down the formula for Wien's displacement law. (1 mark)

(ii) A lamp operates at a filament temperature of 3127°C . Find the minimum frequency of the peak in its blackbody spectrum. (2 mark)

(b) The photoelectric work function for Cesium is 2.14eV .

(i) What is the maximum kinetic energy of the electrons ejected from the surface of Cesium by light of wavelength $\lambda=546\text{nm}$? (2 marks)

(ii) What is the maximum speed of the electrons? (1½ marks)

(iii) Suppose that 1mg of energy is converted to energy and that all the energy is used (with no waste) to operate a 200W lamp. How long can the lamp be operated? (1½ mark)

(c) (i) Define a blackbody. (1 mark)

(ii) A patient waiting to be seen by a physician is asked to remove all his/her clothes in a room whose temperature is 25°C . Calculate the rate of heat loss by radiation from the patient, if his/her skin is at a temperature of 34°C and his/her surface area is 1.6m^2 . Assume an emissivity of 0.8 . (2 marks)

(d) A photon of wavelength 20pm collides with an electron at rest.

(i) Find the Compton wavelength, λ_c , of the scattered electron. (1½ mark)

(ii) Find the wavelength of the photon scattered at an angle of 30° . (2 mark)

(iii) Find the maximum wavelength of the scattered photon. (1½ marks)

(iv) Find the maximum K.E energy of the recoil electrons in electron volts. (2 marks)

(e) Give two applications of X-rays. (2 mark)

Question 3 (20 Marks)

(a) (i) Describe the photoelectric effect. (1 mark)

(ii) Define work function of a material. (1 mark)

(iii) Sketch a well labeled apparatus used to verify the photoelectric effect. (2 marks)

(b) (i) State Bohr's fundamental postulate of the hydrogen atom, regarding the stability of an orbit. (1 mark)

(ii) Consider an electron of mass m , in a hydrogen atom's orbit of radius r , moving at a velocity v . With the help of a diagram, show that the velocity of the electron is

$$v = \sqrt{\frac{e^2}{4\pi\epsilon_0 mr}} \quad (3 \text{ marks})$$

(iii) Find the expression for de Broglie wavelength of an electron orbiting a hydrogen atom. (2 marks)

(iv) Find the de Broglie wavelength of an electron moving in an orbit of radius 1cm. (3 marks)

(c) (i) Distinguish between the production of absorption and emission spectra. (3 marks)

(ii) With the aid of diagrams, illustrate the differences between absorption and emission spectra. (2 marks)

(iii) Explain why condensed matter gives continuous spectra while rarefied gas gives spectra that is characteristic of the atoms or molecules present. (2 marks)

Question 4 (20 Marks)

(a) With the aid of clearly labeled diagrams, distinguish between J.J. Thompson's and Rutherford's models of the atom. (2 marks)

(b) (i) With the aid of a clearly labeled diagram, discuss Rutherford's alpha scattering experiment, which led him to his model of the atom. (5 marks)

(ii) Find the shortest wavelength in the Paschen series of spectral lines. (2 marks)

(c) (i) Define the term impact parameter. (1 mark)

(ii) Sketch a diagram of a charged particle (α) approaching a nucleus, at different values of impact parameter. (2 marks)

(iii) The impact parameter b is related to the scattering angle θ , by the expression,

$$\cot \frac{\theta}{2} = \frac{4\pi\epsilon_0 K}{Ze^2} b, \text{ where the symbols have their usual meaning.}$$

Find the angle through which a 5MeV alpha particle approaching a gold nucleus with an impact parameter of 2.5×10^{-13} cm is scattered. (3 marks)

(d) Starting with the expressions for the angular velocity ω , and the wave number k , show that the phase velocity W , associated with a moving body is $W = \frac{c^2}{V}$, where V is the group velocity and c is the speed of light. (5 marks)

Question 5 (20 Marks)

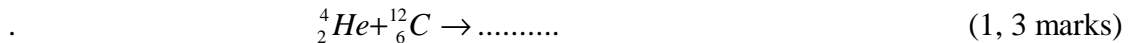
- (a) (i) Distinguish between hard and soft x-rays. (2 marks)
 (ii) Define I. the half life of an isotope. (1 mark)
 II. the mean life time of an isotope. (1 mark)

(iii) Starting with the activity law, show that the half life of an isotope is expressed as,

$$T_{\frac{1}{2}} = \frac{0.693}{\lambda}$$

where symbols have their usual meaning. (2 marks)

- (b) (i) Distinguish between nuclear fission and nuclear fusion. (2 marks)
 (ii) Stars release energy when nuclear reactions take place.
 I. Complete the following nuclear reaction
 II. Find the energy released



(iii) Determine the binding energy per nucleon of a ${}^{238}_{92}\text{U}$ atom. (4 marks)

- (c) Write down the general equations of a radioisotope ${}^A_Z\text{X}$ undergoing the following decay processes . (i) beta decay (1 mark)
 (ii) positron decay (1 mark)
 (iii) electron capture (1 mark)
 (iv) gamma decay (1 mark)