KENYA METHODIST UNIVERSITY
FIRST TRIMESTER 2007 EXAMINATION

FACULTY	**:**	**SCIENCES**
DEPARTMENT	**:**	**MATHEMATICS AND COMPUTER SCIENCE**
COURSE CODE	**:**	**MATH 104**
COURSE TITLE	**:**	**Calculus II**
TIME	**:**	**3 HRS**

Instructions: Attempt Question 1 in **Section A** and any other two questions in **Section B.**

SECTION A
QUESTION 1 (30 Mks)

a) Calculate $\frac{dy}{dx}$ if y = $\int_0^{x^2} \cos tdt$ (2Mks)

b) Evaluate the following integrals

i. $\int \sin(7x+5)dx$ (2mks)

ii. $\int \frac{\cos 2x}{\sin^3 2x}dx$ (3Mks)

c) Let a, b, and m be positive numbers with a<b. Find the area under the graph y = mx, a ≤ x ≤ b. Use inscribed rectangles. (7Mks)

d) Estimate the area under the curve f(x) = 1+x^2 with a = 0, b=1, and n = 4. (5Mks)

e) If f is continuous and F' = f, then
$\int_a^b f(x)dx = F(b) - F(a)$. Prove (5Mks)

f) Suppose f and g are continuous and that
$\int_1^2 f(x)dx = -4$ $\int_2^5 f(x)dx = 6$ $\int_1^5 g(x)dx = 8$

Find
$\int_1^5 f(x)dx$ $\int_5^1 -4f(x)dx$ $\int_1^5 4[f(x) - 2g(x)]dx$ (6Mks)

Section B
Question Two: (20 Mks)

a) Verify the formular (8Mks)

$\sum_{k=1}^{n} k^2 = 1^2 + 2^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}$ for n = 1,2,3. Then add $(n+1)^2$ and thereby prove by mathematical induction that the formular is true for all positive integers.

b) Using the result of (i) show that the area under the graph of y = x^2 over the interval 0≤ x≤ b is $\frac{b^3}{3}$ (6Mks)

c) Calculate the area bounded by the x-axis and the parabola y = 6 – x – x^2 (3Mks)

d) Find the total area bounded by the curve y = x^3 – 4x and the x – axis. (3Mks)

Question Three: (20 Mks)

a) Evaluate the following

(i) $\frac{xdx}{\sqrt{4-x^2}}$ (ii) $\frac{z+1dz}{\sqrt[3]{3z^2+6z+5}}$ (6Mks)

b) Approximate $\int_0^1 4x^3dx$ by the Trapezoidal rule and by Simpson's rule with n = 2. (5Mks)

c) Estimate the error in using (a) the Trapezoidal rule and (b)Simpson's rule to approximate

$\int_1^2 \frac{1}{x}dx$ with n=10. (5Mks)

d) How many subdivisions should be used in the trapezoidal rule to approximate

In2 = $\int_1^2 \frac{1}{x}dx$ with an error of less than 10^{-4} (4Mks)

Question 4: (20 Mks)

a) Let F denote the resultant of all forces acting on a particle of mass m and let the direction of F remain constant. Prove that whether the magnitude of F is constant or variable, the work done on the particle by the force F is W = Δ(K.E) 5Mks)

b) A thin homogenous wire is bent to form a semi – circle of radius r. Find its center of mass. (6Mks)

c) Find the area of the surface obtained by revolving the curve.

$y = \sqrt{x}$, $0 \leq x \leq 2$ about the x- axis. (4Mks)

d) Suppose the above curve is rotated about the x – axis to generate a circle. Find its volume. (2MKs)

e) Find the area bounded by the parabola $y = 2 - x^2$ and the straight line y = -x. (5Mks)