KENYA METHODIST UNIVERSITY

2nd TRIMESTER SCHOOL BASED EXAMINATION
April 2007

FACULTY	**:**	**SCIENCES**
DEPARTMENT	**:**	**MATHEMATICS AND COMPUTER SCIENCE**
COURSE CODE	**:**	**MATH 104**
COURSE TITLE	**:**	**Calculus II**
TIME	**:**	**3 HRS**

Instructions: Attempt Question 1 in **Section A** and any other two questions in **Section B.**

SECTION A

Question 1 (30 Mks)

1. Solve the differential equation

   i. $\frac{dy}{dx} = 3x^2$

2. Evaluate the following integrals

$\int (5x - x^2 + 2)\,dx$ $\int (x^2+5)\,dx$

$\int \cos 2x\,dx$ $\int \sin(7x+5)\,dx$

$\int \frac{\cos 2x}{\sin^3 x}\,dx$

3. Estimate the area under the curve $f(x) = 1+x^2$ with a = 0, b = 1, and n = 4. (use inscribed rectangles)

4. Suppose f and g are continuous and that

$\int_{-1}^{1} f(x)dx = -4$ $\int_{1}^{5} f(x)dx = 6$ $\int_{1}^{5} g(x)dx = 8$

find

$\int_{1}^{5} f(x)dx$ $\int_{5}^{1} -4f(x)dx$ $\int_{1}^{5} [4f(x) - 2g(x)]dx$

5. Find the area under the graph of $y = x^2$, $0 \le x \le$ b. use $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$

SECTION B

Question 2 (20 Mks)

a) Find an antiderivative of each of the following functions **(10mks)**

   i. $F(x) = \cos 6x + 3\sin^2 x$

ii. $G(x) = \sec^2(5x)$
iii. $V(t) = 9\sec 3t \tan 3t$
iv. $W(x) = x^2 + \csc^2 x$
v. $H(x) = e^{2x} + e^{-2x}$

b) Find the derivative of the following functions **(2 mks)**

i. $g(x) = \int_0^x \sqrt{t^4 + t^2 + 1}\,dt$

ii. $f(x) = \int_{-3}^x \frac{(t-2)(t-3)}{t^4 + 16}$

c) Find solutions to the following **(8 mks)**

i. $\int_2^5 (x^2 - x + 3)dx$

ii. $\int_{-1}^1 (x-3)(3x-1)dx$

iii. $\int_{\frac{\Pi}{6}}^{\frac{\Pi}{3}} (2\sin x + \cos x)dx$

iv. $\int_0^2 (x-3)(2x-5)dx$

Question 3 (20 Mks)

Find the following

i. $\int (2x+1)(x^2 + x + 5)^{17} dx$

ii. $\int \frac{2x+5}{\sqrt{x^2 + 5x + 8}}\,dx$

iii. $\int \frac{3x^2}{(x^3 - 1)^5} dx$

iv. $\int 4x(x^2 + 9)^{\frac{5}{2}} dx$

v. $\int \frac{5}{\sqrt{x}(3\sqrt{x}+4)^{\frac{3}{5}}} dx$

vi. $\int x\sec(x^2)\tan(x^2)dx$

vii. $\int x\sin(x^2+1)dx$

viii. $\int \frac{\cos\sqrt{x}}{\sqrt{x}} dx$

ix. $\int \sin^3 x\cos x dx$

x. $\int \sin x\cos^4 x dx$

Question 4 (20 mks)

a) The acceleration of gravity near the surface of the earth is 9.8m/sec^2. This means that the velocity v of a body falling freely in a vacuum changes at the rate of

$$\frac{dv}{dt} = 9.8m/\sec^2$$

If the body is dropped from rest, what will its velocity be t seconds after it is released? **(5 mks)**

b) Evaluate the following **(5 mks)**

i. $\sum_{k=1}^{n}(3k-k^2)$

ii. $\sum_{k=1}^{n}(-a_k)$

iii. $\sum_{k=1}^{3}(k+4)$

iv. $\sum_{k=1}^{4}(k^2-3k)$

c) A heavy projectile is fired straight up from a platform 3m above the ground, with an initial velocity of 160m/sec^2. Assume that the only force affecting the projectile during its flight is from gravity, which produces a downward acceleration of 9.8m/sec^2. Find an equation for the projectile's height above the ground as a function of time t if t = 0 when the projectile is fired. How high above the ground is the projectile 3sec after firing? **(10 mks)**