KENYA METHODIST UNIVERSITY
END OF FIRST TRIMESTER 2007 EXAMINATIONS

FACULTY	**:**	**SCIENCES**
DEPARTMENT	**:**	**MATHEMATICS AND COMPUTER SCIENCE**
COURSE CODE	**:**	**MATH 210**
COURSE TITLE	**:**	**LINEAR ALGEBRA II**
TIME	**:**	**2 HRS**

Instructions:

- Answer question 1 and any other 2 questions.

Question 1

a) Find the matrix representation of the linear map F:$R^2 \longrightarrow R^2$ defined by $F(x,y) = (2x-5y, 3x+y)$ relative to the basis $\{u_1 = (2,1), u_2 = (3,2)\}$ of R^2 (5 mks)

b) Find the coordinates of an arbitrary vector (a,b) in R^2 with respect to the basis $s_1 = \{u_1, u_2\}$, where $u_1 = (1,-2)$, $u_2 = (3,-4)$ (5 mks)

c) Find the cost θ for the angle θ between u = (5,1) and v =(-2, 3) in Euclidean 2-space R^2. In which quadrant does θ lie? (5 mks)

d) Find a unit vector orthogonal to $V_1 = (1,1,2)$ and $V_2 = (0,1,3)$ in R^3. (5 mks)

e) Verify the Cayley-Hamilton theorem for A = $\begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}$ (5 mks)

f) Find the quadratic form q(x,y) corresponding to the symmetric matrix A = $\begin{pmatrix} 5 & -3 \\ -3 & 8 \end{pmatrix}$ (5 mks)

Question 2

a) Consider the vector space V of 2x2 matrices over R and the following usual basis E of V:

$$E = \left\{ E_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, E_2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, E_3 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, E_4 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right\}$$

Let M = $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and T be the linear operator on V defined by T(A) = MA. Find the matrix representation of T relative to the above usual basis of V. (10 mks)

b) Let A = $\begin{pmatrix} 2 & 3 & -4 \\ 4 & -6 & 3 \\ 1 & 4 & -2 \end{pmatrix}$

let B be the matrix representation of the linear map A:$R^3 \longrightarrow R^3$ relative to the basis $\{(1,1,2), (2,3,5), (1,4,6)\}$ Find B. (10 mks)

Question 3

a) Consider two inner products f and g on the same vector space V. Use the functional notation f(u,v) and g(u,v) to denote the inner products of u and v under f and g respectively. Show that the sum f + g, defined by (f+g) (u,v) = f(u,v) + g(u,v) is also an inner product on V. (10 mks)

b) i) suppose M = $\begin{pmatrix} A_1 & B \\ 0 & A_2 \end{pmatrix}$ where A_1 and A_2 are square matrices. Show that the characteristic polynomial of M is the product of the characteristic polynomials of A_1 and A_2 (6 mks)

ii) Generalize the statement in part b(i) above. (4 mks)

Question 4

a) Given the matrix A = $\begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$

i) Compute the eigenvalues and eigenvectors for A. (6 mks)

ii) Determine whether or not a nonsingular matrix P exists such that $P^{-1}AP$ is diagonal. (4 mks)

b) Consider the matrix A = $\begin{pmatrix} 1 & 1 & -1 \\ 1 & 3 & 4 \\ 7 & -5 & 2 \end{pmatrix}$ answer the following with adequate reasons:

i) Are the rows of A orthogonal? (2 mks)

ii) Is A an orthogonal matrix? (2 mks)

iii) Are the columns of A orthogonal? (2 mks)

iv) Let B be the matrix obtained by normalizing each row of A. Find B and whether B is an orthogonal matrix. Are the columns of B orthogonal? (4 mks)

Question 5

a) i) Let f and g be bilinear forms on a vector space V. Show that the sum f+g, defined by (f+g) (u,v) = f(u,v) + g(u,v), is bilinear. (6 mks)

ii) Let f be a bilinear form of a vector space V and let k belong to a field K. Show that the map f, defined by (kf) (u,v) = kf(u,v) , is bilinear. (6 mks)

b) Suppose T is the operator on the vector space V of 2- squares matrices over a field K defined by T(A) = MA where:

$$M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

Find det(T) (8 mks)