KENYA METHODIST UNIVERSITY
END OF SECOND TRIMESTER 2006/2007 EXAMINATIONS

FACULTY : SCIENCES
DEPARTMENT : MATHEMATICS AND COMPUTER SCIENCE
COURSE CODE : MATH 220
COURSE TITLE : ORDINARY DIFFERENTIAL EQUATIONS I
TIME : 3 HRS

Instructions:

- Answer question 1 (compulsory) and any other 2 questions.

Question 1

a) Show that the equation

$(6xy + 2y^2 - 5)dx + (3x^2 + 4xy - 6)dy = 0$ is exact. Hence solve the equation. (6 mks)

b) Solve the Bernoulli equation

$$\frac{dy}{dx} - 2xy = 4xy^{½}$$ (5 mks)

c) Solve the Cauchy-Euler equation

$$3x^2\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + 2y = 0$$ (6 mks)

d) Find the general solution of the equation using the UC method

$$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 5y = 2e^x + 10e^{5x}$$ (6 mks)

e) Find the orthogonal trajectories of the one-parameter family of curves

$$x^2 + y^2 = 2cy$$ (5 mks)

Question 2

a) Solve the equations

i) $$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 13y = 0$$

$y(0) = 0, y^1(0) = -1$

ii) $$\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} + \frac{dy}{dx} - y = 0$$ (8 mks)

b) It is known that a radioactive material decomposes at a rate proportional to the amount present. If after a period of 10 years a 2-gram piece of radium weighs only1.95 grams, how long will it take before the radium is half decayed? (8 mks)

c) Solve the following first order linear equation $(y^2 - xy)dx + x^2dy = 0$ (4 mks)

Question 3

a) Apply the method of exponential shift to solve the differential equation

$$(D^2 - 2D + 5)y = 4x^3e^{3x}$$ (6 mks)

b) Solve the Cauchy-Euler initial value problem

$$x^2\frac{d^2y}{dx^2}-5x\frac{dy}{dx}+8y=2x^3$$

$$y(2)=0, y^1(2)=-8$$ (7 mks)

c) By first finding an integrating factor, solve the equation:

$$(5xy+4y^2+1)dx+(x^2+2xy)dy=0$$ (7 mks)

Question 4

a) An RCL circuit with R = 6 ohms, C = 0.02 farads and L = 0.1 henry, has an applied voltage E(t) = 6 volts. Assuming no initial current and no initial charge when the voltage is first applied, find the subsequent charge on the capacitor in the circuit. (8 mks)

b) Solve the nonhomogenous equation using variation of parameters

$$\frac{d^2y}{dx^2}-4\frac{dy}{dx}+4y=x^{-4}e^{2x}$$

$$y(1)=0, y^1(1)=e^2$$ (7 mks)

c) solve the homogenous first order differential equation.

$$xydx-(x^2+y^2)dy=0$$ (5 mks)