KENYA METHODIST UNIVERSITY
END OF SECOND TRIMESTER 2006/2007 EXAMINATIONS

SCHOOL BASED

FACULTY	**:**	**SCIENCES**
DEPARTMENT	**:**	**MATHEMATICS AND COMPUTER SCIENCE**
COURSE CODE	**:**	**MATH 220**
COURSE TITLE	**:**	**ORDINARY DIFFERENTIAL EQUATIONS I**
TIME	**:**	**2 HRS**

Instructions:

- Answer question 1 (compulsory) and any other 2 questions.

Question 1

a) Define the following terms:
   i) Ordinary differential equations.
   ii) Partial differential equations
   iii) Homogenous first order differential equations (3 mks)

b) Show that the equation

$$(3xy^4 + x)dx + (6x^2y^3 - 2y^3 + 7)dy = 0$$ is exact. Hence find its solution. (6 mks)

c) Solve the Cauchy-Euler initial value problem:

$$x^2\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + 6y = 0$$

(7 mks)

$$y(2) = 0, y^1(2) = 4$$

d) Find the general solution of the equation using the UC method

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^2 - e^x$$ (6 mks)

e) It is known that a radioactive material decomposes at a rate proportional to the amount present. If after a period of 10 years a 2-gram piece of radium weighs 1.95 grams, how long will it take before the radium is half decayed? (8 mks)

Question 2

a) Solve the following equations

i) $\frac{d^2y}{dx^2} - \frac{dy}{dx}$ - 13y = 0

ii) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 30y = 0$ (8 mks)

b) By first finding an integrating factor, solve the equation
$(5xy + 4y^2 + 1)dx + (x^2 + 2xy)dy = 0$ (8 mks)

c) State the uniqueness and existence theorem. (4 mks)

Question 3

a) Solve the Bernoulli equation

$$\frac{dy}{dx} - 2xy = 4xy^{½}$$ (6 mks)

b) Find the orthogonal trajectories of the given family $x^2 + 3y^2 = cy$ (6 mks)

c) Solve the non homogeneous second order equation using variation of parameters method.

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = x^{-4}e^{2x}$$ (8 mks)

Question 4

a) Solve the Cauchy-Euler equation

$$x^2\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 6y = 10x^2$$

$$y(1) = 1, y^1(1) = -6$$ (10 mks)

b) Solve the equation

$$\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} - \frac{dy}{dx} + 2y = 0$$ (5 mks)

c) Solve the homogenous first order differential equation

$$xydx - \left(x^2 + y^2\right)dy = 0$$ (5 mks)