## KENYA METHODIST UNIVERSITY <br> END OF SECOND TRIMESTER 2006/2007 EXAMINATIONS

| FACULTY | $:$ | SCIENCES |
| :--- | :--- | :--- |
| DEPARTMENT | $:$ | MATHEMATICS AND COMPUTER SCIENCE |
| COURSE CODE | $:$ | MATH 221 |
| COURSE TITLE | $:$ | VECTOR ANALYSIS |
| TIME | $:$ | 2 HRS |

## Instructions:

- Answer question 1 (compulsory) and any other 2 questions.


## Question 1

a) If $\underline{A}$ and $\Phi$ are a vector point function and a scalar point respectively find :
i) $\quad \operatorname{grad} \Phi \quad$ (ii) $\quad$ curl $\underline{A}$ at the point $(1,1,1)$
b) Evaluate $\iint_{5} \underline{F} d \underline{S}$ given that
$\underline{F}=(x-2) \underline{i}+(x+3 y) \underline{j}+2 z \underline{k}$ over the closed surface of the tetrahedron formed by the planes $\mathrm{x}=0$,
$\mathrm{y}=0$ and $2 \mathrm{x}+2 \mathrm{y}+\mathrm{z}=2$
c) Evaluate $\int_{c} \underline{F} . d \underline{r}$ if $\underline{F}=x \underline{i}-y \underline{j}+z \underline{k}$ and C is the straight line segment from $(1,1,1)$ to $(-2,2,3)$
d) Find the angle between the vectors

$$
\begin{equation*}
\underline{A}=3 \underline{i}-\underline{j}+2 \underline{k} \text { and } \underline{B}+2 \underline{i}+\underline{j}-\underline{k} \tag{5mks}
\end{equation*}
$$

e) Prove that the line joining the mid points of a triangle is parallel to the third side and has one half of its magnitude.

## Question 2

a) State Green's Theorem in the plane.
b) Verify Green's Theorem in the plane for:

$$
\begin{equation*}
\oint_{c}\left(x y+y^{2}\right) d x+x^{2} d y \tag{15mks}
\end{equation*}
$$

Where C is the closed curve of the region bounded by $\mathrm{y}=\mathrm{x}$ and $\mathrm{y}=\mathrm{x}^{2}$

## Question 3

Evaluate

$$
\int_{v} \underline{F} d v \text { where } \mathrm{V} \text { is the region bounded by the planes } \mathrm{x}=0, \mathrm{y}=0, \mathrm{z}=0 \text { and } 2 \mathrm{x}+\mathrm{y}+\mathrm{z}=2 \text { and }
$$

$$
\begin{equation*}
\underline{F}=2 Z \underline{i}+y \underline{k} \tag{20mks}
\end{equation*}
$$

## Question 4

Verify the divergence theorem for the vector field $\underline{F}=x^{2} \underline{i}+z j+y \underline{k}$ taken over the region bounded by the planes $\mathrm{z}=0, \mathrm{z}=2, \mathrm{x}=0, \mathrm{x}=1, \mathrm{y}=0, \mathrm{y}=3$

