KENYA METHODIST UNIVERSITY
END OF SECOND TRIMESTER 2006/2007 EXAMINATIONS

FACULTY : SCIENCES
DEPARTMENT : MATHEMATICS AND COMPUTER SCIENCE
COURSE CODE : MATH 331
COURSE TITLE : OPERATIONS RESEARCH I
TIME : 3 HRS

Instructions:
- Answer question 1 (compulsory) and any other 2 questions in section B.

Question 1 (30 marks)

a) State and explain briefly three main basic elements of a mathematical model in operations research. (3 mks)

b) Define the following:
   i) a feasible solution
   ii) an optimal solution (2 mks)

c) Ozark farms uses at least 800 kg of special feed daily. The special feed is a mixture of corn and soybean meal with the following composition.

Feed stuff	protein	Fiber	Cost (Sh/Kg)
Corn	0.09	0.02	30
Soybean	0.60	0.06	90

The dietary requirements of the special feed stipulate at least 30% protein and atmost 5% fiber. Ozark farms wishes to determine the daily minimum cost feed mix.
   i) Form a linear optimization model. (4 mks)
   ii) Use graphical method to solve the linear programming model in (i) above. (5 mks)

d) Write the following linear programme in standard form

$$\text{Minimize } x_0 = 10x_1 + 5x_2 + 20x_3$$
$$\text{Subject to : } x_1 + x_2 + 2x_3 \geq 10$$
$$2x_1 + x_2 + 3x_3 \geq 20$$
$$x_2 + 2x_3 = 5$$

x_1 is unrestricted, $x_2, x_3, \geq 0$ (5 mks)

e) Write the dual of the following linear programme:

$$\text{Max } z = 10x_1 + 20x_2 + 15x_3 + 40x_4$$
$$\text{Subject to } x_1 + 2x_2 + 4x_4 \geq 10$$
$$2x_1 + x_2 + 3x_3 + x_4 \geq 30$$
$$x_2 + x_3 + 2x_4 \leq 10$$

$x_{1,} x_2, x_4 \geq 0, x_3$ is unrestricted. (5 mks)

f) Consider the following linear programming problem:

$$\text{Max } x_0 = 3x_1 + 2x_2 + 5x_3$$
$$\text{Subject to } x_1 + 2x_2 + x_3 \leq 430$$
$$3x_1 + 2x_3 \leq 460$$
$$x_1 + 4x_3 \leq 420$$
$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$$

Obtain the standard form of this programme and find the optimum solution. (6 mks)

Question 2 (20 marks)

a) State the three main properties of a general linear programming problem. (3 mks)

b) Consider the following linear programming problem:

Minimize $x_0 = 20x_1 + 30x_2 + 50x_3 + 40x_4$

Subject to
$$4x_1 + 6x_2 + x_3 + 2x_4 \geq 12$$
$$2x_1 + x_2 + 6x_3 + 5x_4 \geq 14$$
$$x_1 + 2x_2 + 4x_3 + 3x_4 \geq 8$$

$$x_i \geq 0, i = 1,2,3,4$$

i) Write the complete dual of the above primal problem. (5 mks)

ii) Use simplex method to find the optimal solution of the primal problem by solving the dual in (i) above. (12 mks)

Question 3 (20 marks)

a) Given the linear program

Minimize $z = 4x_1 + x_2$
$$3x_1 + x_2 = 3$$
$$4x_1 + 3x_2 \geq 6$$
$$x_1 + 2x_2 \leq 4$$
$$x_1 \geq 0, x_2 \geq 0$$

Apply the M-technique to solve the above linear programme. (14 mks)

b) Use graphical method to solve the following linear program.

Maximize $z = 5x_1 + 4x_2$
$$6x_1 + 4x_2 \leq 24$$
$$x_1 + 4x_2 \leq 6$$
$$-x_1 + 2x_2 \leq 1$$
$$x_2 \leq 2$$

$$x_1 \geq 0, x \geq 0$$ (6 mks)

Question 4 (20 marks)

a) Define a degenerate solution. (2 mks)

b) What are the possible implications of degeneracy in a linear program? (4 mks)

c) Solve the following linear programme using simplex method.

Max $Z = 3x_1 + 9x_2$

Subject to
$$x_1 + 4x_2 \leq 8$$
$$x_1 + 2x_2 \leq 4$$
$$x_1 \geq 0, x_2 \geq 0$$ (14 mks)