KENYA METHODIST UNIVERSITY

END OF TRIMESTER EXAM APRIL 2008

FACULTY	**:**	**SCIENCE AND SOCIAL STUDIES**
DEPARTMENT	**:**	**COMPUTER INFORMATION SCIENCE**
COURSE CODE	**:**	**MATH 310**
COURSE TITLE	**:**	**REAL ANALYSIS I**
MODE	**:**	**SCHOOL BASED**

Total Marks (60)

TIME: 2 ½ HOURS

Instructions

Answer all questions in SECTION A and ANY ONE question in SECTION B

Question 1 (30 Mks)

1. Define the following terms
   a. Point of contact.
   b. A set is utmost countable.
   c. Cardinality of a set. (3 marks)
2. State the completeness axiom of real numbers R. (2 marks)
3. Show that every open neighbourhood is an open set. (5 marks)
4. Show that a subset of metric space (x,d) is open if and only if $A = A^o$.

   (6 marks)

5. Let (X_n) be a monotonic sequence of real numbers. Prove that (X_n) converges if and only if it is bounded. (5 marks)
6. Let (X,ρ) be a metric space and $E \subseteq \mathbf{X}$. Define the notions:
   a. E is bounded.
   b. A limit point of E.
   c. Interior of E in the metric space (X,ρ).
   d. Neighbourhood of a point in the metric space (X,ρ). (4 marks)
7. Show that every subset of E of X is open as well as closed in X. (5 marks)

Question 2 (30 marks)

1. Examine for extreme values the function

$$f(x,y) = e^{2x + 3y}(8x^2 - 6xy + 3y^2) \qquad \text{(15 marks)}$$

2. Show that the interval (0,1) is equivalent to $\mathbf{R}$. That is, card $\mathbf{R}$ = Card (0,1) (5 marks)

3. Show that

$\frac{x}{x+1} + \frac{x}{(x+1)(2x+1)} + \frac{x}{(2x+1)(3x+1)} + \ldots +\ldots$, is uniformly convergent in (k, ∞) when k is any positive integer. Show also that the series is not uniformly convergent near the point x=0.

(10 marks)

Question 3 (30 marks)

1. Show that $\sqrt{2}$, a root if $x^2 = 2$ is an irrational number. (4 marks)

2. Prove that for all x,y $\in \mathbf{R}$

$| |x| - |y|| \leq |x-y|$. (4 marks)

3. Consider (R, d), show that the function f: R $\rightarrow \mathbf{R}$ defined by f(x) = |x| is uniformly continuous. (5 marks)

4. Show that every convergent sequence is Cauchy. (5 marks)

5. If $f(x) = \begin{cases} 1+2x & \text{for f rational} \\ 0 & \text{for f irrational} \end{cases}$

Calculate $\int_0^1 f(x)dx$ (6 marks)

6. If a and b are positive and p is any positive integer, show that as n $\rightarrow \infty$

$\sum_{r=1}^{np} \frac{1}{na+r} \rightarrow \log\left(1+\frac{p}{a}\right)$ (3 marks)

7. Show that a set is infinite if and only if it is equivalent to a proper subset of itself. (3 marks)

Question 4 (30 marks)

1. Let x be a nonempty set and defines a function d(x,y) for all x,y elements of x by

$d(x,y) = \begin{cases} 1 if x \neq y \\ 0 if x = y \end{cases}$ Show that the function d is a metric. (8 marks)

2. Let f:X$\rightarrow$Y be a continuous function and X be a compact metric space. Prove that the range of f, f(x) is also compact. (8 marks)

3. Let E = (a,d] $\cup$ {c} where c>b be a subset of the matrix space (R, d). find the derived set of E, E^d. (5 marks)

4. Show that the subset A in a metric space (x,d) is closed if and only if A = $\overline{A}$.

(5 marks)

5. Explain the following terms of a metric space (x,d_1) and (y,d_2):

   a. Continuity of the function f.

   b. Limit of the function f. (4 marks)