KENYA METHODIST UNIVERSITY

END OF FIRST TRIMESTER 2008 EXAMINATIONS

FACULTY	**:**	**SCIENCE AND SOCIAL STUDIES**
DEPARTMENT	**:**	**COMPUTER AND INFORMATION SCIENCE**
COURSE CODE	**:**	**MATH 110**
COURSE TITLE	**:**	**LINEAR ALGEBRA I**
TIME	**:**	**2 HOURS**

Instructions:

- *Answer question **ONE** (compulsory) and any other **TWO** questions.*

<u>Question 1 (30 marks)</u>

a) (i) Solve the system

$$x_1 + 2x_2 = 5$$
$$-x_1 + x_2 = 1 \qquad (3 \text{ mks})$$
$$x_1 + x_2 = 6$$

(ii) Using the augmented matrix approach, find

$$A^{-1} \ if \ A\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} \qquad (3 \text{ mks})$$

b) (i) Solve the system A$\underline{x}$ = $\underline{b}$ where

$$\text{A}=\begin{bmatrix} 1 & -2 & 3 \\ 3 & 0 & 1 \\ 2 & 2 & 1 \end{bmatrix}, \ \underline{b}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}. \qquad (3 \text{ mks})$$

(ii) show that if the vectors $\underline{v}_1, \underline{v}_2$ *and* $\underline{v}_3$ are independent then the vectors $\underline{u}_1 = 2\underline{v}_1, \underline{u}_2 = \underline{v}_1 + \underline{v}_2$ *and* $\underline{u}_3 = \underline{v}_3 - \underline{v}_1$ are also independent. (3 mks)

c) Let V be the subset of $\Re^3$ consisting of all vectors $\underline{v} = (v_1, v_2, v_3)$ such that $v_3 = 2v_1$. Determine whether V is a subspace of $\Re^3$. (5 mks)

d) (i) Define an inner product of a vector space $\Re^m$. (3 mks)
(ii) Determine whether the vectors (1,0,0), (2,2,4,), (-1,0,1) span $\Re^3$. (3 mks)

e) (i) Find a basis for $\Re^5$ that contains the vectors $\underline{v}_1 = (1,1,0,0,0)$, $\underline{v}_2 = (1,1,1,1,0)$.
(4 mks)
(ii) If V is a subspace spanned by $\underline{v}_1, \underline{v}_2, \underline{v}_3$ find an orthornomal basis for v. (3 mks)

Question 2 (20 mks)

a) Consider the transformation $T:\Re^2 \to \Re^3$ defined by
$T(x_1, x_2) = (x_1 + x_2, 3x_2, 2x_1 - x_2)$. Prove that T is a linear transformation. (10 mks)

b) Consider a linear transformation $T:\Re^3 \to \Re^2$ defined by
$T(x_1, x_2, x_3) = (x_1 - x_2, x_2 + x_3)$.
Find the matrix of T with respect to the basis
$\underline{v}_1(2,0,1), \underline{v}_2 = (0,2,2)$ *and* $\underline{v}_3 = (0,2,3)$ *of* $\Re^3$ and the basis
$\underline{w}_1 = (1,2)$ *and* $\underline{w}_2 = (0,1)$ *of* $\Re^2$ (10 mks)

Question 3 (20 marks)

a) Let V be a subspace of $\Re^4$ consisting of vectors $\underline{x} = (x_1, x_2, x_3, x_4)$ such that $x_1 + x_2 - x_3 + x_4 = 0$. Find a basis for V that contains the vector (0,0,1,1). What is dim V?

b) Show that if A is an n x n matrix, the subset $\Re^n$ consisting of all solution vectors of the homogeneous system A$\underline{x}$ = $\underline{0}$ is a subspace of $\Re^n$ called the nullspace of A. (10 mks)

Question 4 (20 marks)

a) (i) Let $A = \begin{bmatrix} 2 & 1 \\ 0 & 1 \\ 2 & 1 \end{bmatrix}$, $\underline{\delta}_1 = \begin{bmatrix} 3 \\ -1 \\ 3 \end{bmatrix}$ and $\underline{\delta}_2 = \begin{bmatrix} 3 \\ 1 \\ 8 \end{bmatrix}$.

Determine whether $\underline{\delta}_1$ and $\underline{\delta}_2$ are in the column space of A. (7 mks)

(ii) Prove that if $\underline{v}_1, \underline{v}_2, ..., \underline{v}_t$ are non-zero orthogonal vectors in a vectors space $\Re^m$, then they are linearly independent. (6 mks)

b) Let $\underline{u} = (1,2,5)$ *and* $\underline{v} = (5,1,1)$ find the distance between $\underline{u}$ and $\underline{v}$. Express $\underline{w}$ = (-7, 4, 13) as a linear combination of $\underline{u}$ and $\underline{v}$. (7 mks)

Question 5 (20 marks)

a) Show that

(i) The vectors $\underline{x}_1 = (1,1,1), \underline{x}_2 = (1,1,0)$ and $\underline{x}_3 = (1,0,0)$ span $\Re^3$ (5 mks)

(ii) The polynomials $f_1(t) = t^2 + 2t - 1$, $f_2(t) = t + 1$, $f_3(t) = 1$ span the space of polynomials of degree ≤ 2. (5 mks)

b) (i) Write the vector $\underline{z} = (-2,1,3)$ as a linear combination of $\underline{x}_1 = (1,1,1)$, $\underline{x}_2 = (1,1,0)$ *and* $\underline{x}_3 = (1,0,0)$. (5 mks)

(ii) Prove that a linear transformation is one-to-one if and only if its kernel is trivial that is $\ker(T) = \{\underline{0}\}$. (5 mks)