KENYA METHODIST UNIVERSITY

1st TRIMESTER EXAMINATION
Jan - April 2008

FACULTY	**:**	**SCIENCE AND SOCIAL STUDIES**
DEPARTMENT	**:**	**COMPUTER & INFORMATION SCIENCE**
COURSE CODE	**:**	**MATH 221**
COURSE TITLE	**:**	**VECTOR ANALYSIS**
MODE	**:**	**SCHOOL BASED**
TIME	**:**	**2 Hrs**

Instructions: Answer Question 1 and other two Questions

Question 1 (20 Marks)

1. If $\mathbf{r}_1 = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$, $\mathbf{r}_2 = 2\mathbf{i} - 4\mathbf{j} - 3\mathbf{k}$, $\mathbf{r}_3 = -2\mathbf{i} + \mathbf{j} - 3\mathbf{k}$ and $\mathbf{r}_4 = 3\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}$, find scalars a,b,c such that $\mathbf{r}_4 = a\mathbf{r}1 + b\mathbf{r}_2 + c\mathbf{r}_3$. **(5 Mks).**

2. Forces **A, B** and **C** acting on an object are given in terms of their components by the vector equations $\mathbf{A} = A_1\mathbf{i} + A_2\mathbf{j} + A_3\mathbf{k}$, $\mathbf{B} = B_1\mathbf{i} + B_2\mathbf{j} + B_3\mathbf{k}$, $\mathbf{C} = C_1\mathbf{i} + C_2\mathbf{j} + C_3\mathbf{k}$. Find the magnitude of these forces. **(5 Mks).**

3. The position vectors of points P and Q are given by $\mathbf{r}_1 = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$, $\mathbf{r}_2 = 4\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$. determine **PQ** in terms of **i, j, k** and find its magnitude. (**5 Mks)**

4. If **A** is any vector, prove that $\mathbf{A} = (\mathbf{A}.\mathbf{i})\mathbf{i} + (\mathbf{A}.\mathbf{j})\mathbf{j} + (\mathbf{A}.\mathbf{k})\mathbf{k}$ **(5 Mks)**

Question 2 (20 Marks)

1. A particle moves along a curve whose parametric equations are
   $x = e^{-t}, y = 2\cos 3t, z = 2\sin 3t$ where *t* is the time.

   i. Determine its velocity and acceleration at any time
   ii. Find the magnitudes of the velocity and acceleration at t = 0

   (5 Mks)

2. i)Find the unit tangent vector to any point on the curve $x = t^2+1$, $y = 4t - 3$, $z = 2t^2 - 6t$
   ii)Determine the unit tangent at the point where t = 2. **(5 Mks)**

3. If $\mathbf{A} = 5t^2\mathbf{i} + t\mathbf{j} - t^3\mathbf{k}$ and $\mathbf{B} = \sin t\mathbf{i} - \cos t\mathbf{j}$ find

i) $\frac{d}{dt}(\mathbf{A}.\mathbf{B})$ **ii)** $\frac{d}{dt}(\mathbf{A}\times\mathbf{B})$ **iii)** $\frac{d}{dt}(\mathbf{A}.\mathbf{A})$ **(5 Mks)**

4. Determine a unit vector that is perpendicular to the plane of $\mathbf{A} = 2\mathbf{i} - 6\mathbf{j} - 3\mathbf{k}$ and $\mathbf{B} = 4\mathbf{i} + 3\mathbf{j} - \mathbf{k}$. Similarly determine a unit vector parallel to the same. (**5 Mks**)

Question 3 (20 Marks)

1. If $\phi(x, y, z) = 3x^2y - y^3z^2$ find $\nabla\phi$ (or grad ϕ) at the point (1,-2,-1). **(3 Mks)**

2. Find a unit normal to the surface $x^2y + 2xz = 4$ at the point (2,-2,3). **(2 Mks)**

3. Find the directional derivative of $\phi = x^2yz + 4xz^2$ at (1,-2,-1) in the direction $2\mathbf{i} - \mathbf{j} - 2\mathbf{k}$ **(5 Mks)**

4. If $\mathbf{A} = xz^3\mathbf{i} - 2x^2yz\mathbf{j} + 2yz^4\mathbf{k}$ find ∇xA (or curl **A)** at the point (1, -1, 1) **(5 Mks)**

5. If $\mathbf{R}(\underline{u}) = (\underline{u - u^2})\mathbf{i} + 2u^3\mathbf{j} - 3\mathbf{k}$ find

i) $\int R(u)du$ ii) $\int_1^2 R(u)du$ **(5 mks)**

Question 4 (20 Marks)

1. If $\mathbf{A} = (3x^2 + 6y)\mathbf{i} - 14yz\mathbf{j} + 20xz^2\mathbf{k}$, evaluate $\int_c A.dr$ from (0,0,0) to (1,1,1) along the following paths C:

   i. $x = t, y = t^2, z = t^3$
   ii. The straight lines from (0,0,0) to (1,0,0), then to (1,1,0), then to (1,1,1).
   iii. The straight line joining (0,0,0) and (1,1,1). **(5 Mks)**

2. Find the area of the triangle having vertices at P(1,3,2), Q(2,-1,-1), R(-1,2,3). **(5 Mks)**

3. If $\mathbf{A} = A_1\mathbf{i} + A_2\mathbf{j} + A_3\mathbf{k}$, $\mathbf{B} = B_1\mathbf{i} + B_2\mathbf{j} + B_3\mathbf{k}$, $\mathbf{C} = C_1\mathbf{i} + C_2\mathbf{j} + C_3\mathbf{k}$ show that

$$\mathbf{A}.(\mathbf{B} \times \mathbf{C}) = \begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix}$$ **(5 Mks)**

4. For what values of a are $\mathbf{A} = a\mathbf{i} - 2\mathbf{j} + \mathbf{k}$, $\mathbf{B} = 2a\mathbf{i} + a\mathbf{j} - 4\mathbf{k}$ perpendicular? **(5 Mks)**