KENYA METHODIST UNIVERSITY

END OF FIRST TRIMESTER 2008 EXAMINATIONS

FACULTY	**:**	**SCIENCE AND SOCIAL STUDIES**
DEPARTMENT	**:**	**COMPUTER AND INFORMATION SCIENCE**
COURSE CODE	**:**	**MATH 332**
COURSE TITLE	**:**	**THEORY OF ESTIMATION**
TIME	**:**	**2 HOURS**

Instructions:

- *Answer question **ONE** (compulsory) and any other **TWO** questions.*

<u>Question 1</u>

a) Explain the four properties of a good estimator. (8 mks)

b) Let $x_1, x_2, ..., x_n$ be a random sample selected from a population with mean μ and variance σ^2. Show that the sample variance $s^2 = \frac{1}{n-1}\sum\left(x-\bar{x}\right)^2$ is an unbiased estimator for σ^2 (5 mks)

c) Let $x_1, x_2, ..., x_n$ be a random sample from a normal distribution with mean μ and variance σ^2 with p.d.f given as $f\left(x,\mu,\sigma^2\right) = \frac{1}{\sigma\sqrt{2\pi}}\exp\left\{-\frac{1}{2\sigma}\left(x-\mu\right)^2\right\}$
Find the maximum likelihood estimators for μ and σ^2. (7 mks)

d) The following sample of IQ measurements of students selected from a class whose distribution is normal

91	80	99	110	95	106	78	121	106
100	97	82	100	83	115	104	114	118
96	101	79	130	101	94	101	79	

calculate
(i) Standard error. (2 mks)
(ii) 98% confidence interval for the true mean. (5 mks)

e) Explain what is meant by Minimum Variance Unbiased Estimators (MVUE). (5 mks)

<u>Question 2</u>

a) A random sample of n observations $x_1, x_2, ..., x_n$ is selected from a population that possesses a Gamma probability density function with parameters α *and* β.
$f\left(x,\alpha,\beta\right) = \frac{1}{\Gamma\alpha\beta^{\alpha}} e^{-x/\beta} x^{\alpha-1}, \; x \geq 0$.
Where $\mu = \alpha\beta \;\; and \sigma^2 = \beta^2$
Find the moment estimators for the unknown parameters. (10 mks)

b) State Crammer-Rao inequality and give its significance in the theory of estimation. (4 mks)

c) The Binomial distribution is given as

$$f(x, p) = \binom{n}{x} p^x (1-p)^{n-x}, \; x = 0,1,\ldots,n.$$

Show that the distribution belong to the one parameter exponential families. Hence or otherwise find a complete and sufficient statistic for p. (6 mks)

Question 3

a) Briefly explain what is meant by Bayesian Estimation. (4 mks)

b) Let $y = \beta_0 + \beta_1 x + \varepsilon$ be a linear model and ε is normally distributed with mean 0 and variance σ^2. The least squares estimators are given as

$$\hat{\beta}_1 = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2}$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

Show that the estimators $\hat{\beta}_1$ $and \hat{\beta}_0$ are unbiased for β_1 and β_0 respectively.(8 mks)

c) The IQ scores given below were obtained at random from a normally distributed population.
85, 91, 93, 99, 103, 107, 111, 115, 122, 95 construct a 95% confidence interval for the population variance. (8 mks)

Question 4

a) State Rao-Blackwell Theorem. (3 mks)

b) Given a sample $x_1, x_2, \ldots, x_n$ is distributed as Bernoulli
$f(x/p) = p^x (1-p)^{1-x}, \; x = 0,1$ and the distribution of the parameter p is uniform
$h(p) = 1, \; 0 \le p \le 1$
Find the posterior distribution of p.

c) Show that the normal distribution with mean μ and variance σ^2 belong to a two parameter exponential families. (5 mks)

d) The following sample was selected from a normally distributed with unknown mean μ and known σ^2
69, 81, 67, 80, 71, 70, 78, 68, 57, 59
Construct a 98% confidence interval for the population mean. (7 mks)