# KENYA METHODIST UNIVERSITY

**END OF FIRST TRIMESTER 2008 EXAMINATIONS**

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| **FACULTY** | **:** | **SCIENCE AND SOCIAL STUDIES** |
| **DEPARTMENT** | **:** | **COMPUTER AND INFORMATION SCIENCE** |
| **COURSE CODE** | **:** | **MATH 332** |
| **COURSE TITLE** | **:** | **THEORY OF ESTIMATION** |
| **TIME** | **:** | **2 HOURS** |

**Instructions:**

- *Answer question **ONE** (compulsory) and any other **TWO** questions.*

**Question 1**

a) Explain the four properties of a good estimator. (8 mks)

b) Let $x_1, x_2, ..., x_n$ be a random sample selected from a population with mean μ and variance $\sigma^2$. Show that the sample variance $s^2 = \frac{1}{n-1}\sum\left(x - \bar{x}\right)^2$ is an unbiased estimator for $\sigma^2$ (5 mks)

c) Let $x_1, x_2, ..., x_n$ be a random sample from a normal distribution with mean μ and variance $\sigma^2$ with p.d.f given as $f\left(x, \mu, \sigma^2\right) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{1}{2\sigma}(x-\mu)^2\right\}$

Find the maximum likelihood estimators for μ and $\sigma^2$. (7 mks)

d) The following sample of IQ measurements of students selected from a class whose distribution is normal

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 91 | 80 | 99 | 110 | 95 | 106 | 78 | 121 | 106 |
| 100 | 97 | 82 | 100 | 83 | 115 | 104 | 114 | 118 |
| 96 | 101 | 79 | 130 | 101 | 94 | 101 | 79 | |

calculate

(i) Standard error. (2 mks)

(ii) 98% confidence interval for the true mean. (5 mks)

e) Explain what is meant by Minimum Variance Unbiased Estimators (MVUE). (5 mks)

**Question 2**

a) A random sample of n observations $x_1, x_2, ..., x_n$ is selected from a population that possesses a Gamma probability density function with parameters $\alpha$ *and* $\beta$.

$f\left(x, \alpha, \beta\right) = \frac{1}{\Gamma\alpha\beta^\alpha} e^{-x/\beta} x^{\alpha-1}, \ x \geq 0$.

Where $\mu = \alpha\beta \ \ and \sigma^2 = \beta^2$

Find the moment estimators for the unknown parameters. (10 mks)

b) State Crammer-Rao inequality and give its significance in the theory of estimation. (4 mks)

c) The Binomial distribution is given as

$$f(x, p) = \binom{n}{x} p^x (1-p)^{n-x},\ x = 0,1,\ldots,n.$$

Show that the distribution belong to the one parameter exponential families. Hence or otherwise find a complete and sufficient statistic for p. (6 mks)

**Question 3**

a) Briefly explain what is meant by Bayesian Estimation. (4 mks)

b) Let $y = \beta_0 + \beta_1 x + \varepsilon$ be a linear model and $\varepsilon$ is normally distributed with mean 0 and variance $\sigma^2$. The least squares estimators are given as

$$\hat{\beta}_1 = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2}$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1\bar{x}$$

Show that the estimators $\hat{\beta}_1$ *and* $\hat{\beta}_0$ are unbiased for $\beta_1$ *and* $\beta_0$ respectively.(8 mks)

c) The IQ scores given below were obtained at random from a normally distributed population.
85, 91, 93, 99, 103, 107, 111, 115, 122, 95 construct a 95% confidence interval for the population variance. (8 mks)

**Question 4**

a) State Rao-Blackwell Theorem. (3 mks)

b) Given a sample $x_1, x_2, \ldots, x_n$ is distributed as Bernoulli
$f(x/p) = p^x(1-p)^{1-x},\ x = 0,1$ and the distribution of the parameter p is uniform
$h(p) = 1,\ 0 \le p \le 1$
Find the posterior distribution of p.

c) Show that the normal distribution with mean $\mu$ and variance $\sigma^2$ belong to a two parameter exponential families. (5 mks)

d) The following sample was selected from a normally distributed with unknown mean $\mu$ and known $\sigma^2$
69, 81, 67, 80, 71, 70, 78, 68, 57, 59
Construct a 98% confidence interval for the population mean. (7 mks)