KENYA METHODIST UNIVERSITY
End of Trimester I, 2008 Examination

Faculty	**:**	**Science and Social Studies**
Department	**:**	**Computer and Information Science**
Course Code	**:**	**MATH 410**
Course Title	**:**	**Complex Analysis I**
Mode	**:**	**School-Based**
Time	**:**	**2 Hours**

Instructions: Answer Question One and ANY OTHER TWO questions

Question One (30 marks) – Compulsory:

a. Give two complex numbers U = 3 + 5i and V = 5 – 7i, determine:
   (i.) U+ V
   (ii.) V * U (4 marks)

b. Define the domain of definition for the function $f(z) = \frac{1}{z^2 + 1}$ (4 marks)

c. Verify that $\frac{1 + 2i}{3 - 4i} + \frac{2-i}{5i} = \frac{-2}{5}$ (6 marks)

d. Use de Moivre's formula to derive the following trigonometric identity:
   $\text{Cos}3\theta = \text{Cos}^3\theta - 3\text{Cos}\theta\text{Sin}^2\theta$ (6 marks)

e. Find all the tree cube roots of -8i (6 marks)

f. Write the function $f(z) = z + 1/z$ $(z \neq 0)$ in the form $f(z) = u(r,\theta) + iv(r,\theta)$ (4 marks)

Question Two (20 marks)

a. Show that the function $f(z) = z^2$ is differentiable (4 marks)

b. Use the properties of conjugates and moduli to show that
   i. $\overline{\bar{z} + 3i} = z + 3i$ (6 marks)
   ii. $|z| = |\bar{z}|$ (4 marks)

c. Given $f(z) = \frac{2z + 1}{z(z^2 + 1)}$
   Determine the singular points and state why the function is analytic everywhere except at those points (6 marks)

Question Three (20 marks)

a. Using Cauchy-Riemann equations, show that f'(z) does not exist if $f(z) = 2x + ixy^2$ (4 marks)

b. Show that $|e^{i\theta}| = 1$ (5 marks)

c. Find the principle argument Arg Z when $z = \frac{-2}{1 + \sqrt{3}i}$ (5 marks)

d. Evaluate the integrals of t, for the following functions, in the given intervals:
   i. $F(z) = e^{-zt}$, 0 to ∞
   ii. $F(z) = e^{it}$, 0 to π/4 (6 marks)

Question Four (20 marks)

a. Find the four roots of the equation $z^4 + 4 = 0$ and use them factor $z^4 + 4$ into quadratic factors with real coefficients (10 marks)

b. Describe the domain of definition for the function $f(z) = \frac{1}{z^2 + 1}$ (4 marks)

c. Express the function $f(z) = z^3$ in polar coordinates (4 marks)

d. Solve $(1-i)^4$ (2 marks)