# KENYA METHODIST UNIVERSITY

# END OF TRIMESTER EXAM APRIL 2009FACULTY:FACULTY:COURSE CODE:COURSE TITLE:COURSE TITLE:SIMULATION AND MODELING

# Total Marks (60) TIME: 2 HOURS

### **Instructions**

Answer all questions in SECTION A and ANY TWO question in SECTION B

### **SECTION A – Answer all questions**

### **Question 1 – 30 marks**

- a. Define the following terms:
  - a. Simulation clock
    - b. Confidence interval (2 marks)
- b. Outline three standard capabilities of simulation languages (3 marks)
- c. Consider the following graph that illustrates the time-dependent queue of a simulation of a single-server system, which utilizes the random variables shown.



- i. Calculate the average customer delay in the queue. (3 marks)
- ii. Calculate the expected average queue length. (3 marks)

- iii. Calculate the expected utilization of the server. (3 marks)
- d. Suppose that the 10 observations 1.20, 1.50, 1.68, 1.89, 0.95, 1.49, 1.58, 1.55, 0.50 and 1.09 are from a normal distribution with unknown mean *m* and that our objective is to construct a 90% confidence interval for *m*.

i.	Calculate the sample mean and the standard deviation of the 10					
	observations.	(3 marks)				
ii.	Calculate the confidence interval for <i>m</i> .	(3 marks)				

- e. State four reasons that can make you use simulation. (4 marks)
- f. Using the four digit positive integer  $Z_0 = 7182$ , describe the mid-square method for generating random numbers. (4 marks)
- g. List two desirable qualities of a good random number generator.

(2 marks)

### **SECTION B – Answer ANY TWO questions**

### **Question 2 – 15 marks**

A small supermarket is trying to determine how many copies of *People* magazine they should order each week. They believe their demand for *People* is governed by the following discrete random variable:

Dem	and Probability	
15	0.10	
20	0.20	
25	0.30	
30	0.25	
35	0.15	

The supermarket pays \$1.00 for each copy(which is stored in cell C4) of *People* and sells it for \$1.95(which is stored in cell C5). Each Un-sold copy can be returned for \$0.50((which is stored in cell C6).

Assume that you were to carry out a Monte Carlo simulation in excel to determine how many copies of People magazine the store should order.

Assume also that cells F2:G6 is named as *myvalues*.

Assume that the value to be produced is in cell C1,

Write down the **formulas only**, which will perform the following computations:

- i. Create a random number in cell C2. (1 mark)
- ii. In cell C3, simulate the demand for the magazine. (2 marks)

- iii. In cell C8, calculate the revenue. (2 marks)
  iv. In cell C9, calculate the total production cost. (2 marks)
  v. In Cell C10, calculate the total disposing cost. (2 marks)
- vi. In Cell C11, calculate the profit. (2 marks)
- vii. Assume that you have made 1000 iterations starting from cell A17, and you have already simulated the demand for each production quantity in cells B18 to E1017, and that the production quantities from B17 to E17 are 15, 20, 25 and 30 respectively:
  - 1. Starting from cell B15 for production quantity 15, calculate the average simulated profit for each production quantity. (1 mark)
  - 2. Starting in cell B16 for production quantity 15, calculate the standard deviation for each production quantity. (1 mark)
- viii. Calculate the lower and upper 95% confidence interval on mean profit when 25 magazines are produced. (1 marks)

# Question 3 – 15 marks

a. The following crude arrival interval time data (time between successive arrivals) were acquired through observational sampling:

1, 6, 2, 1, 4, 3, 4, 2, 1, 8, 2, 1, 5, 1, 1, 2, 1.

- (i) Calculate the average, variance and standard deviation on this data. (3 marks)
- (ii) Based on these data, provide a corrected (t distribution) estimate of the uncertainty of the average at the 95% confidence interval.

(4 marks)

- b Consider the LCG defined by m=16, a=5, c=3 and  $Z_0$ =7. Generate 5 random numbers, U<sub>1</sub> to U<sub>5</sub>. (5 marks)
- c For a given family or continuous distributions there are usually several ways to define, or parameterize, the probability density function. Describe these ways.

(3 marks)

## **Question 4 – 15 marks**

- a State five advantages of programming a simulation model in a simulation language rather than a general-purpose language (5 marks)
- b A queuing system is characterized by three components. Name and describe these components. (3 marks)
- c Suppose we conduct 10 independent simulation runs to estimate the mean throughput of a network system. The observed throughputs from these 10 simulation runs are 7.3, 6.1, 3.8, 8.4, 6.9, 7.1, 5.3, 8.2, 4.9 and 5.8.
  - i) Calculate the mean, variance and standard deviation. (2 marks)

ii) What is the 95% confidence interval if we use the Normal distribution model?

(2marks)

iii) What is the 99% confidence interval if we use the t-distribution model? (3marks)

df	80%	90%	95%	99%	df	80%	90%	95%	99%
1	3.078	6.314	12.706	63.657	16	1.337	1.746	2.120	2.921
2	1.886	2.920	4.303	9.925	17	1.333	1.740	2.110	2.898
3	1.638	2.353	3.182	5.841	18	1.330	1.734	2.101	2.878
4	1.533	2.132	2.776	4.604	19	1.328	1.729	2.093	2.861
5	1.476	2.015	2.571	4.032	20	1.325	1.725	2.086	2.845
6	1.440	1.943	2.447	3.707	21	1.323	1.721	2.080	2.831
7	1.415	1.895	2.365	3.500	22	1.321	1.717	2.074	2.819
8	1.397	1.860	2.306	3.355	23	1.319	1.714	2.069	2.807
9	1.383	1.833	2.262	3.250	24	1.318	1.711	2.064	2.797
10	1.372	1.812	2.228	3.169	25	1.316	1.708	2.060	2.787
11	1.363	1.796	2.201	3.106	26	1.315	1.706	2.056	2.779
12	1.356	1.782	2.179	3.055	27	1.314	1.703	2.052	2.771
13	1.350	1.771	2.160	3.012	28	1.313	1.701	2.048	2.763
14	1.345	1.761	2.145	2.977	29	1.311	1.699	2.045	2.756
15	1.341	1.753	2.131	2.947	30	1.310	1.697	2.042	2.750

# **Student's t-Table**

df	80%	90%	95%	99%
infinity	1.282	1.645	1.96	2.576