KENYA METHODIST UNIVERSITY

END OF FIRST TRIMESTER EXAMINATIONS 2009

FACULTY	:	ARTS AND SCIENCES
DEPARTMENT	:	COMPUTER INFORMATION SYSTEMS
COURSE CODE	:	MATH 103
COURSE TITLE	:	CALCULUS I
TIME	:	3 HOURS

Instructions:

- *Answer question* ***ONE*** *and any other* ***TWO*** *questions.*

Question 1 (30 marks)

a) Evaluate the following limits.

(i) $\lim_{x \to 2} \dfrac{x^2 - 5x + 6}{x^2 - 3x - 10}$

(ii) $\lim_{x \to 0} \dfrac{\sin 2x}{3x}$ (5 marks)

b) Differentiate $\dfrac{x}{x^2 - 2}$ from the first principles. (5 marks)

c) Find an approximate value for the square root of 63.5 (5 marks)

d) If $y = \cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right)$, show that $\dfrac{dy}{dx} = \dfrac{2}{1+x^2}$ (6 marks)

e) A wire of length 42cm is bent into a rectangle of length *x* cm. Express the area, *A* as a function of *x*. Hence calculate the length of the rectangle for which the area is a maximum and determine this maximum area. (5 marks)

f) Differentiate $y = \log_e\left(\sin 2x - x^3\right)$ (5 marks)

Question 2 (20 marks)

A function *f(x)* is defined as follows.

$$f(x) = \begin{cases} -2x+1 & \text{if } x < -1 \\ x+4 & \text{if } -1 \le x < 1 \\ 5 & \text{if } 1 < x < 2 \\ 3 & \text{if } x \ge 2 \end{cases}$$

a) Sketch the function on the Cartesian plane (5 marks)

b) State any points of discontinuity of $f(x)$ (1 mark)

c) Evaluate the following limits:
(i) $\lim_{x \to -1} f(x)$ (ii) $\lim_{x \to 1^-} f(x)$, (iii) $\lim_{x \to 2} f(x)$ (3 marks)

d) A curve is defined parametrically by $x = \frac{2t}{t+2}$ and $y = \frac{3t}{t+3}$.
Find the equation of the tangent through the point $(1.2, 1.5)$ (7 marks)

e) Find $\lim_{x \to 4} \frac{\sqrt{5x-4} - 2^2}{x-4}$ (4 marks)

Question 3 (20 marks)

a) A conical flask of base radius 10 cm and slant angle 75^0 is collecting water at a uniform rate of 100 cubic cm per second. At what rate will the radius of the water surface be changing when the radius of the water surface is 5 cm? (9 marks)

b) Given the curve $y = 3x^4 - 4x^3$, find
i. The x-intercepts (1 mark)
ii. The stationary points and state their nature (5 marks)
iii. Sketch the curve (1 mark)

c) Find the points on the graph $f(x) = \frac{x+1}{x^2+8}$ at which the tangent line is horizontal.
(4 marks)

Question 4 (20 marks)

a) State the three conditions that a continuous function $f(x)$, must satisfy at a point $x = a$. (3 marks)

b) Given the function $f(x) = \frac{3x^2 - 13x - 10}{2x^2 - 11x + 5}$
i. Show that $\lim_{x \to 5} f(x)$ is indeterminate.
ii. By redefining $f(x)$, show that the point of discontinuity is removable. (7 marks)

c) A particle starts from a point A and moves through a distance s metres in t seconds governed by the equation $s = 5t^2 - 18t + 16$.
i. Determine the times when the particle is at A.
ii. Show that its acceleration is a constant and state its value.
iii. Calculate the velocity
- Initially
- After 3 seconds (6 marks)

d) Evaluate $\lim_{x \to +\infty} \frac{5.5^x - 3^x.3^2}{2(5^x) + 3^x}$ and $\lim_{x \to -\infty} \frac{5.5^x - 3^x.3^2}{2(5^x) + 3^x}$ (4 marks)