KENYA METHODIST UNIVERSITY

END OF 1ST TRIMESTER 2009 EXAMINATIONS

FACULTY	:	ARTS AND SCIENCES
DEPARTMENT	:	COMPUTER INFORMATION SYSTEMS
UNIT CODE	:	MATH 110
UNIT TITLE	:	LINEAR ALGEBRA I
TIME	:	2 HOURS

Instructions:

- *Answer question ONE and any other TWO questions.*

Question 1 (30 marks)

a) Determine the values of a so that the following system in unknowns x, y and Z has:

i) No solution

ii) More than one solution

iii) A unique solution

$x + y - z = 1$

$2x + 3y - az = 3$

$X + ay - 3z = 2$ (7 mks)

b) Find the inverse of the matrix using row reduction method.

$$A = \begin{bmatrix} -1 & 2 & -3 \\ 2 & 1 & 0 \\ 4 & -2 & 5 \end{bmatrix}$$ (6 mks)

c) Determine whether or not the following system of vectors is linearly dependent.
(3,1,2), (2,0,6), (4,1,4) (5 mks)

d) Suppose U and W are subspaces of a vector space V. Show that U + W is a subspace of V. (5 mks)

e) Let W be the subspace of R^4 generated by the vectors {(1, -2, 5, -3), (2, 3, 1, -4), (3, 8, -3, -5)}. Find a basis and the dimension of W. (7 mks)

Question 2 (20 marks)

a) Let $T:R^3 \rightarrow R^3$ be the linear transformation defined by

$T(x, y, z) = (z + 2y - z, y + z, x + y - 2z)$

Find a basis and the dimensions of:

i) Kernel of T

ii) Image of T (8 mks)

b) Use Cramer's rule to solve the system of equations:

$2x_1 + 4x_2 + 6x_3 = 18$
$4x_1 + 5x_2 + 6x_3 = 24$
$3x_1 + x_2 - 2x_3 = 4$ (9 mks)

c) Calculate the distance of the plane $2x - 5y + 3z + 8 = 0$ from origin. (3 mks)

Question 3 (20 marks)

a) Find the rank and basis of the matrix:

$$B = \begin{bmatrix} 1 & 3 & 1 & -2--3 \\ 1 & 4 & 3 & -1 & -4 \\ 2 & 3 & -4 & -7 & -3 \\ 3 & 8 & 1 & -7 & -8 \end{bmatrix}$$ (7 mks)

b) Let $F:R^2 \to R^2$ be the linear transformation for which F(3,1) = (2-4) and F(1,1) = (0,2)

i) Find the formula for F

ii) Find F(7,4) (7 mks)

c) Find the value of K for the vector (1, -2, k) to be a linear combination of vectors (3, 0, -2) and (2, -1, -5). (6 mks)

Question 4 (20 marks)

a) Show that the matrix is idempotent. $E = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ (5 mks)

b) Determine whether S = {(2, 1, -1, 0), (1, 2, 1, 3), (-1, -2, 1, -2), (2, 1, 1, -1)} is a basis of R^4. (7 mks)

c) Find the inverse of the matrix by first finding the adjoint.

$$B = \begin{bmatrix} 1 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$$

Hence solve:

$x_1 + \quad x_3 = 2$
$x_1 - 2x \quad = -1$
$\quad 2x_2 + x_3 = 1$ (8 mks)