KENYA METHODIST UNIVERSITY
END OF ${ }^{1 \pi}$ TRIMESTER 2009 EXAMINATIONS

| FACULTY | $:$ | ARTS AND SCIENCES |
| :--- | :--- | :--- |
| DEPARTMENT | $:$ | COMPUTER INFORMATION SYSTEMS |
| UNIT CODE | $:$ | MATH 211 |
| UNIT TITLE | $:$ | DISCRETE STRUCTURES |
| TIME | $:$ | 2 HOURS |

## Instructions:

- Answer question ONE and any other TWO questions.


## Question 1 (30 marks)

a) Show that all terms of the sequence $u n=2 .\left(4^{n}\right)+1$ are divisible by 3. ( 5 mks )
b) A certain family consists of a mother, father, daughter and two sons. The family members have influences (or power) over each other in the following ways: the mother can influence the daughter and the oldest son; the father can influence the two sons; the daughter can influence the father; the oldest son can influence the youngest son and the youngest son can influence the mother.
Model this family influence pattern with a directed graph and write down the corresponding vertex matrix.
c) Use Boolean algebra to simplify $\mathrm{Au}(\mathrm{A}$ 'nb) and represent the statement in a Venn diagram.
d) Find the number of permutations for all the letters in MISSISSIPI.
e) Write down the first three terms in the expansion of $(1+2 x)^{11}$. Hence calculate $1.02^{10}$ correct to 2 significant figures.
f) Two players R and C have each a stationery wheel with a movable pointer. R's wheel (row-wheel) is divided into three sectors, namely $1,2,3$ with respective angles $60^{\circ}$, $120^{\circ}$ and $180^{\circ}$, while C's wheel (column wheel) is divided into 4 sectors, namely 1,2 , 3,4 with respective angles $90^{\circ}, 90^{\circ}, 120^{\circ}$ and $60^{\circ}$. Depending on the move each player makes, player $C$ then makes a payment of money to player $R$ according to the following table.

|  | Player C's move |  |  |  |  |
| :---: | :--- | :--- | :--- | :--- | :--- |
|  | 1 | 2 | 3 | 4 |  |
| Player R's <br> move | 1 | $£ 3$ | $£ 5$ | $-£ 2$ | $-£ 1$ |
|  | 2 | $-£ 2$ | $£ 4$ | $-£ 3$ | $-£ 4$ |
|  | 3 | $£ 6$ | $-£ 5$ | $£ 0$ | $£ 3$ |

How much should $R$ expect to receive from player in each play of the game? ( 6 mks )
g) An urn contains 5 white balls and 4 black balls. 2 balls are drawn from the urn. Find the probability that the balls taken are both black.

## Question 2 (20 marks)

a) State the fundamental principle of counting.
b) How many even numbers greater than 2, 000 can be formed with the digits 1, 2, 4, 8 if each digit may be used only once in each number?
c) A committee of 4 is to be selected from 5 members of party $A$ and five member of party B.
i) How many possible committees are there? (6 mks)
ii) In how many of these will party A have a majority?
(3 mks)
d) Simplify $\frac{21!}{7!4!}+\frac{21!}{8!3!}$
(4 mks)

## Question 3 (20 marks)

a) Two competing television networks, R and C are scheduling 1-hour program in the same time period. Network R can schedule one of three possible programs and network $C$ can schedule one of four possible programs. Neither network knows which program the other will schedule. Both networks ask the same outside polling agency to give them an estimate of how all possible pairings of the programme will divide the viewing audience. The agency gives them each, the following table whose ( $\mathrm{i}, \mathrm{j}$ ) this entry is the percentage of the viewing audience that will watch network R. If network R's program is paired against network C's program.

|  | Network C's Program |  |  |  |  |
| :---: | :--- | :--- | :--- | :--- | :--- |
|  | 1 | 2 | 3 | 4 |  |
| Network R's <br> Program | 1 | 60 | 20 | 30 | 55 |
|  | 2 | 50 | 75 | 45 | 60 |
|  | 3 | 70 | 45 | 35 | 30 |

i) What program should each network schedule in order to maximize its viewing audience?
ii) What percentages of the audience will the two networks receive?
b) A die is tossed and a card drawn from cards labeled 1, 2, 3, 4, 5.
i) Draw the possibility space for this scenario.
ii) Find the probability of obtaining:
i. A double
ii. A double or a sum greater than 7
iii. A difference of 3 given a sum greater than $y$.

## Question 4

a) Use mathematical induction principle to prove that:

$$
\begin{equation*}
1^{2}+3^{2}+s^{2}+\ldots+(2 n-1)^{2}=\frac{1}{3} n\left(4 n^{2}-1\right) \tag{7mks}
\end{equation*}
$$

b) Find the term independent of x in the expansion of $\left(\frac{x^{2}}{2}-\frac{2}{x}\right)^{4}$
c) Use Boolean algebra to prove De Morgan's law (AuB’) = A’nB'
d) An urn contains 5 white balls and 4 black balls. Two balls are drawn from the urn, one after the other without replacement. Find the probability that:
i) the balls taken are both black.
ii) balls of different colors are drawn.

