KENYA METHODIST UNIVERSITY
FIRST TRIMESTER EXAMINATION
APRIL 2009

FACULTY	**:**	**ARTS & SCIENCES**
DEPARTMENT	**:**	**COMPUTER INFORMATION SYSTEMS**
COURSE CODE	**:**	**MATH 310**
COURSE TITLE	**:**	**REAL ANALYSIS I**
TIME	**:**	**2HRS**
MODE	**:**	**SCHOOL BASED**

Instructions: Attempt Question 1 and any other two questions.

SECTION A

QUESTION 1 (30 Mks)

a) Determine whether each of the following series converges. In each case, justify your answer

i. $\sum \frac{n}{n^3 + n + 1}$ [3]

ii. $\sum \frac{(-1)^n}{n^3 + n + 1}$ [3]

iii. $\sum \frac{n!(2n)!}{(3n)!}$ [3]

iv. $\sum \frac{(-1)^{n+1}}{\sqrt{n} + \log(\sqrt{n} + 1)}$ [3]

b) What does it mean to say that a series is "conditionally convergent"?

c) Suppose that the series $\sum a_n$ is conditionally convergent, and that b_n c_n sequences and are defined as follows:

$b_n = \max(a_n, 0) = \frac{1}{2}(a_n + |a_n|)$,

$c_n = \min(a_n, 0) = \frac{1}{2}(a_n - |a_n|)$

Prove that $\sum b_n$ and $\sum c_n$ both diverge. (you may use, without proof, the fact that if $\sum x_n$ and $\sum y_n$ converge, then for any $\alpha, \beta \in \Re, \sum (\alpha x_n + \beta y_n)$ converges. [10]

c) State the Mean Value Theorem. [3]

d) i)What is meant by an open subset of $\Re^n$?
ii)What is meant by a closed subset of the same? [5]

Question 2 (20 Marks)

a) Show that the following series converges for any a>1:

$$\sum \frac{n}{(n^2+1)(\log(n^2+1))^a}$$ [7]

b) Determine the values of the real number x for which the following series converges:

$$\sum \frac{nx^n}{n^2+2}$$ [6]

c) Suppose that $a_n \geq 0$ and that the series $\sum a_n$ diverges. Prove that the series

$\sum \frac{a_n}{1+a_n}$ also diverges. [7]

Question 3 (20 marks)

a) The function f:$R^2\backslash\{(0,0)^T \to \Re$ is given, for x = $(x,y)^T$,by $f(x) = \frac{x^2}{x^6+y^2}$

Prove that the $\lim_{x\to 0} f(x)$ does not exist. (Here, 0 = (0,0)T [10]

b) Prove that if a function $f:\Re \to \Re$ has a local minimum at a point at which it is differentiable, then the derivative at that point is 0. [10]

Question 4 (20 Marks)

a)Suppose that $f:\Re \to \Re$ is differentiable and that $|f'(x)| < 1$ for all x. Prove that there can be utmost one solution of the equation f(x) = x. [10]

b)Suppose that C is a subset of R^n. State the formal definition of what it means for C to be compact. Using this formal definition, prove that the union of two compact subsets of R^n is itself a compact subset. [10]