## KENYA METHODIST UNIVERSITY

END OF $1^{s T}$ TRIMESTER 2009 EXAMINATIONS

| FACULTY | $:$ | ARTS AND SCIENCES |
| :--- | :--- | :--- |
| DEPARTMENT | $:$ | COMPUTER INFORMATION SYSTEMS |
| UNIT CODE | $:$ | MATH 331 |
| UNIT TITLE | $:$ | OPERATIONS RESEARCH I |
| TIME | $:$ | 2 HOURS |

## Instructions:

- Answer question ONE (compulsory) and any other TWO questions.


## Question 1

a) Define the following terms:
i) Feasible solution
ii) Slack variable
iii) Surplus variable
b) State three main properties of a general linear programming problem. (3 mks)
c) A firm produces two products A and B using two machines 1 and 2. One unit of product $A$ requires 2 hours on machine 1 and 2 hours on machine 2. One unit of product $B$ requires 3 hours on machine 1 and 1 hour on machine 2. Machine 1 is available for 12 hours per day while machine 2 is available for 8 hours per day. If the profits are Ksh. 600 per unit of product $A$ and Ksh. 800 per unit of product $B$.
i) Form a linear optimization model.
ii) Use graphical model to solve the linear programming model in (i) above.
d) Obtain the dual of the following primal linear programme.

Minimize $x_{0}=30 x_{1}+40 x_{2}+12 x_{3}+8 x_{4}-3 x_{5}$
Subject to: $\quad 3 x_{1}+x_{2}+x_{3}-x_{5} \geq 20$

$$
x_{1}+3 x_{2}+x_{3}+x_{4} \geq 10
$$

$$
\begin{equation*}
x_{i} \geq 0, I=1,2,3,4,5 \tag{5mks}
\end{equation*}
$$

## Question 2 (20 marks)

A company has 1000 tonnes of ore $x, 2000$ tonnes of ore $y$ and 500 tonnes of $z$. products A, $B$ and $C$ are to be produced from these ores. For one tone of each of these produces the ore requirements are listed below:

|  | X | Y | Z | Profit (Ksh) per <br> tone |
| :--- | :--- | :--- | :--- | :--- |
| A | 5 | 10 | 10 | 100 |
| B | 5 | 8 | 5 | 200 |
| C | 10 | 5 | 0 | 50 |
| Max available resources | 1000 | 2000 | 500 |  |

i) Applying the simplex algorithm, calculate how many tones of each of the products $\mathrm{A}, \mathrm{B}$ and $C$ should the company produce so as to maximize its profits.
ii) Evaluate this profit.

## Question 3 ( 20 mks )

a) Consider the linear programming functions given below:
maximize $z=2 \times 1+x 2 \leq 10$

$$
\begin{array}{ll}
\text { subject to: } & x_{1}+5 x_{2} \leq 10 \\
& x_{1}+3 x_{2} \leq 6 \\
& 2 x_{1}+2 x_{2} \leq 8 \\
& x_{1}, x_{2}=\geq 0
\end{array}
$$

Find:
i) The solutions that maximize $z$ using simplex method.
ii) The equivalent dual function
b) Apply the $M$-technique to find the solution to the following linear program.

Minimize $z=4 x_{1}+x_{2}$
Subject to:

$$
\begin{aligned}
& 3 x_{1}+x_{2}=3 \\
& 4 x_{1}+3 x_{2} \geq 6 \\
& x_{1}+2 x_{2} \leq 4 \\
& x_{1} \geq 0, x_{2} \geq 0
\end{aligned}
$$

## Question 4 (20 marks)

a) State and explain briefly three main basic elements of a mathematical model in operations research.
b) Apply the cutting plane method to solve the following intergar linear programme.

Maximize $z=7 \times 1+10 \times 2$

Subject to

$$
\begin{aligned}
& -x_{1}+3 x_{2} \leq 6 \\
& 7 x_{1}+x_{2} \leq 35 \\
& x_{1}, x 2 \geq 0 \text { and integer }
\end{aligned}
$$

