KENYA METHODIST UNIVERSITY

END OF 2ND TRIMESTER 2010 EXAMINATIONS

NYERI CAMPUS

SCHOOL : SCIENCE & TECHNOLOGY
DEPARTMENT : COMPUTER SCIENCE AND BUSINESS INFORMATION
UNIT CODE : MATH 103
UNIT TITLE : CALCULUS I
TIME : 2 HOURS

Instructions: Attempt Question 1 and any other two questions.

Question 1 – 30 marks

a) Find the following limits

i. $\lim_{x \to 3} \frac{x^2 - 8x + 15}{x^2 + 4x - 21}$ **(3Mks)**

ii. $\lim_{x \to -3} \frac{x^3 + 27}{x^2 - 3x - 18}$ **(3Mks)**

iii. $\lim_{x \to 2} \frac{x^3 - 8}{x^2 - 9x + 14}$ **(3Mks)**

iv. $\lim_{x \to -4} \frac{x^2 + 3x - 4}{x^3 + 64}$ **(3 Mks)**

b) Find the derivatives of the following

i. $g(x) = (3x^2 - 5)(4x + 1)$ **(3 Mks)**

ii. $f(x) = \frac{2x + 5}{3x^2 - 4}$ **(3 Mks)**

c) Use the chain rule to solve the following

$A(x) = (x^2 - 11x + 23)^{\frac{7}{3}}$ **(3Mks)**

d) Prove that $\frac{d}{dx}(\sin x) = \cos x$ **(5 Mks).**

e) Study the continuity question of each of the following functions f near the given point *a*, and comment.

i. $f(x) = \begin{cases} x^2 + 9, if x < -1, \\ 3x + 13, if x \geq -1 \end{cases}$ a = -1 **(2 Mks)**

ii. $f(x) = \frac{3x - 15}{x^2 - 25}; a = 5$ **(2 Mks)**

SECTION B – Answer ANY TWO questions

Question 2 – 20 marks

a) Solve the following using L'hopital's rule **(3 Mks)**

$$\lim_{x \to \infty} \frac{3x^2 - x + 8}{2x^2 + 5x + 7}$$

b) Find the derivatives of the following

$g(x) = (3x^2 - 5)(4x + 1)$ **(3 Mks)**

$w(x) = (x^2 + 4)\sin x + (3x - 5)\cos x$ **(3 Mks)**

c) A free fall of a heavy ball bearing is released from rest at time t = 0 sec.

i. How many meters does the ball fall in the first 2 sec? **(2 Mks)**

ii. What is its velocity, speed, and acceleration then? **(3 Mks)**

d) Prove that $\frac{d}{dx}(\sin x) = \cos x$ **(6 Mks)**

Question 3 – 20 marks

a) Find the slope of the circle $x^2 + y^2 = 25$ at the point (3, -4). **(4 Mks)**

b) Find the tangent and the normal to the curve $x^2 - xy + y^2 = 7$
at the point (-1,2). **(6 Mks)**

c) Suppose that y is a function of x that satisfies the equation $x^5 + y^5 = 32$ and suppose that y''

exists. Find y''. **(4 Mks)**

d) State and prove the Mean Value Theorem **(6 Mks)**

Question 4 – 20 marks

a) Find the critical points of $f(x) = x^{\frac{1}{3}}(x - 4) = x^{\frac{4}{3}} - 4x\frac{1}{3}$ **(3 Mks)**

b) Find the intervals on which $g(x) = -x^3 + 12x + 5, -3 \le x \le 3$ **(5 Mks)**

c) Find two positive numbers whose sum is 20 and whose product is as large as possible **(5 Mks)**

d) Find *dy if* **(3 Mks)**

i. $y = x^5 + 37x$

ii. $y = sin3x$

e) An open-top box is to be made by cutting small congruent squares from the corners of a 12-by-12-inch sheet of tin and bending up the sides. How large should the squares cut from the corners be to make the box hold as much as possible? **(4 Mks)**