KENYA METHODIST UNIVERSITY

END OF 2ND TRIMESTER 2010 EXAMINATIONS

SCHOOL   :   SCIENCE & TECHNOLOGY
DEPARTMENT :   COMPUTER SCIENCE AND BUSINESS INFORMATION
UNIT CODE   :   MATH 104
UNIT TITLE   :   CALCULUS II
TIME   :   2 HOURS

INSTRUCTIONS TO CANDIDATES:

- *Answer __QUESTION 1__ and __ANY OTHER TWO__ questions.*

QUESTION ONE (30 MARKS)

a) Define the fundamental theorem of integral calculus.

Hence evaluate $\int_1^2 x^2 dx$ (5 marks)

b) Use integral calculus to show that the volume V of a cone, base radius r and height, h is

$V = \frac{1}{3}\pi r^2 h$ (4 marks)

c) Evaluate $\int \frac{x^3 + 1}{x^3 - x} dx$ 6 marks)

d) Find the area between the curves $y = 4x$ and $y = 2x^2$ (4 marks)

e)   Evaluate $\frac{d}{dx}\int_x^{x^2} \frac{\sin t}{t} dt$ (4 marks)

f) Calculate the length round the circle, centre O, radius r (7 marks)

QUESTION TWO (20 MARKS)

a) Consider the region in the xy plane bounded by $y = 4 - x^2$ and the $x - axis$.

   i.   Make a sketch of this region (2 marks)

   ii.   Find the area of the region. (5 marks)

   iii.   Find the moment of inertia about the $y - axis$ (*Assume unit density*) (5 marks)

b) Given the force $\vec{F} = xy\,\hat{i} - y^2\,\hat{j}$, show that $dw = xydx - x^2dy$ and hence find the work done by

$\vec{F}$ along the straight path joining the origin (0,0) to the point $(2,1)$ (8 marks)

QUESTION THREE (20 MARKS)

a) Find the length of the arc of the parabola $y = x^2$ from $x = 0$ to $x = 1$ (8 mks)

b) Find the area generated when the arc of the parabola $y = 8x$ between $x = 0$ and $x = 2$ is rotated about the x-axis. (6 marks)

c) Calculate the area between the curves $y = 2x - 1$ and $y = x^2 - 1$ (6 marks)

QUESTION FOUR (15 MARKS)

a) Prove the reduction formula

$$I_n = \int \sin^n x \, dx = \frac{n-1}{n} I_{n-2} - \frac{\cos x \sin^{n-1} x}{n}.$$
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(5 marks)

Hence evaluate $\int \sin^6 x \, dx$ (4 marks)

b) Evaluate $\int \dfrac{dx}{1 + \cos x}$ (5 marks)

c) $\int_0^{\frac{\pi}{2}} \sin^5 x \cos^2 x \, dx$ (6 marks)

QUESTION FIVE (15 MARKS)

a) Define an improper integral. (2 marks)

b) Evaluate $\int_{-1}^{0} \sqrt{\dfrac{1+x}{1-x}} \, dx$ (8 marks)

c) Evaluate $\int_0^2 \dfrac{dx}{\sqrt{16 - x^2}}$ (3 marks)

d) A particle moves such that at any given time t, its velocity is $v = 6t^2 - 26t + 22 \ ms^{-1}$. If after one second the particle is 3 metres from a given fixed point O, find the times the particle is at O. (7 marks)