KENYA METHODIST UNIVERSITY

END OF 2ND TRIMESTER 2010 EXAMINATIONS

FACULTY	**:**	**SCIENCE AND TECHNOLOGY**
DEPARTMENT	**:**	**COMPUTER SCIENCE & BUSINESS INFORMATION**
UNIT CODE	**:**	**MATH 110**
UNIT TITLE	**:**	**LINEAR ALGEBRA 1**
TIME	**:**	**2 HOURS**

INSTRUCTIONS TO CANDIDATES:

- *Answer **QUESTION 1** and **ANY OTHER TWO** questions.*

QUESTION ONE (30 MARKS)

a) Express $(2.1.1)$ as a linear combination of $(1,0,2)$, $(0,0,1)$, $(-1,-1,1)$ (4 Marks)

b) Reduce to echelon form the augmented matrix for the system of equations below and hence deduce that the system has no solution

$$\begin{aligned} 2x + y + 3z &= 6 \\ x + y + 4z &= 1 \\ x - z &= 1 \end{aligned}$$

(4 Marks)

c) Verify whether or not the following system of vectors is linearly dependent: $(3,1,2)$, $(2,0,6)$ $(4,1,4)$ (5 Marks)

d) If $\vec{v}_1 = (1,0,1,0)$ $\vec{v}_2 = (0,2,0,1)$ $\vec{v}_3 = (1,0,-1,0)$, show that $\vec{v}_1$, $\vec{v}_2$ and $\vec{v}_3$ is an orthogonal set and find a vector $\vec{v}_4$ in $\Re^4$ such that $\vec{v}_1$, $\vec{v}_2$, $\vec{v}_3$ and $\vec{v}_4$ is an orthogonal basis of $\Re^4$ (8 Marks)

e) Calculate the distance of the plane $2x - 5y + 3z + 8 = 0$ from the origin (3 Marks)

f) If $E = \{(x, y) : x + y + z = 0)\}$, determine whether E is a subspace of $\Re^3$. (6 Marks)

QUESTION TWO (20 MARKS)

a) Determine whether or not the following form a basis for the vector space $\Re^3$:

i. $(1,1,1)$ and $(1,-1,5)$ (1 Mark)

ii. $(1,2,3)$, $(1,0,-1)$ $(3,-1,0)$ and $(2,1,-2)$ (1 Mark)

iii. $(1,1,1)$, $(1,2,3)$ and $(2,-1,1)$ (4 Marks)

iv. $(1,1,2)$, $(1,2,5)$ and $(5,3,4)$ (4 Marks)

b) Given the following basis of Euclidean space $\Re^3$ $\left\{ \vec{v}_1 = (1,1,1),\ \vec{v}_2 = (0,1,1),\ \vec{v}_3 = (0,0,1) \right\}$ use Gram –Schmidt orthogonalization process to transform $\left\{ \vec{v}_i \right\}$ into an orthogonal basis $\left\{ \vec{u}_i \right\}$ (10 Marks)

QUESTION THREE (20 MARKS)

a) Use Cramer's rule to solve the system of equations (7 Marks)

$$\begin{aligned} 2x_1 + 4x_2 + 6x_3 &= 18 \\ 4x_1 + 5x_2 + 6x_3 &= 24 \\ 3x_1 + x_2 - 2x_3 &= 4 \end{aligned}$$

b) Given the linear system

$$\begin{aligned} 2x_1 + 3x_2 - 4x_3 &= 5 \\ - 4x_2 + 2x_3 &= -8 \\ x_1 - x_2 + 5x_3 &= 9 \end{aligned}$$

Obtain the inverse of the coefficients matrix and hence solve the system of equations. (13 Marks)

QUESTION FOUR (20 MARKS)

Given the points $A = (1,1,1)$ $B = (2,0,2)$ and $C = (-1,1,2)$, find

a) the equation of the plane through A, B and C (7 Marks)
b) the angle between AB and AC (7 Marks)
c) the distance from $P(2,2,6)$ to the plane passing through A, B and C (6 Marks)

QUESTION FIVE (20 MARKS)

a) State the conditions necessary for a linear function to have an inverse. (2 Marks)

If $f(w) = (3x,+2y,\ x-3z,\ y+4z)$, find vectors $\vec{u}$, $\vec{v}$ $\vec{w}$ so that $f(\vec{u}) = (1,0,0)$, $f(\vec{v}) = (0,1,0)$ $f(\vec{w}) = (0,0,1)$ and hence find a formula for $f^{-1}(x,y,z)$ (13 Marks)

b) Find the rank and basis of the matrix (5 Marks)

$$A = \begin{pmatrix} 1 & 3 & 1 & -2 & -3 \\ 1 & 4 & 3 & -1 & -4 \\ 2 & 3 & -4 & -7 & -3 \\ 3 & 8 & 1 & -7 & -8 \end{pmatrix}$$