



MUEO

# MOI UNIVERSITY

OFFICE OF THE CHIEF ACADEMIC OFFICER

## UNIVERSITY EXAMINATIONS

### 2010/2011 ACADEMIC YEAR

#### THIRD YEAR SECOND SEMESTER EXAMINATION

## FOR THE DEGREE OF BACHELOR OF ENGINEERING IN ELECTRICAL & COMMUNICATIONS ENGINEERING

**COURSE CODE:** ECE 342

**COURSE TITLE:** TRANSMISSION LINES

**DATE:** 28<sup>TH</sup> APRIL, 2011      **TIME:** 9.00 A.M. – 12.00 NOON.

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### INSTRUCTION TO CANDIDATES

- SEE INSIDE

THIS PAPER CONSISTS OF (5) PRINTED PAGES

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## INSTRUCTION TO CANDIDATES

1. ANSWER ANY FIVE QUESTIONS
2. SMITH CHARTS ARE PROVIDED

## QUESTION 1 ✓

(a) Using suitable diagrams, describe THREE types of transmission lines that are commonly in use. (3 marks) ✓

(b) Explain the following terms that are used to describe transmission lines

(i) Uniform

(ii) Loss-less

(iii) Lossy

(iv) Distortionless

(2 marks) ✓

(c) Draw an equivalent circuit of a section of transmission line of length  $\Delta z$  which has distributed parameters  $R\Omega/m$ ,  $LH/m$ ,  $GS/m$  and  $CF/m$  (3 marks) ✓

(d) Apply Kirchoff's voltage and current laws to the equivalent circuit, and show that for time harmonic signals

$$\frac{\partial^2 I}{\partial z^2} - (R + j\omega L)(G + j\omega C)I = 0$$

(4 marks) ✓

R.L.S.C.

## QUESTION 2 ✓

(a) A 300m long line has the following distributed parameters:  
 $R=10.4\Omega/\text{Km}$ ,  $L=3.67\text{mH}/\text{Km}$ ,  $C=8.35\text{nF}/\text{Km}$  and  $G=0.8\mu\text{S}/\text{Km}$ .

Calculate the:

(i) attenuation constant ✓

(ii) phase constant ✓

(iii) characteristic impedance ✓

(iv) wavelength and ✓

(v) velocity of propagation at a frequency of 796Hz. (10 marks)

(b) The line in (a) above is terminated by its characteristic impedance. A 2V generator of internal impedance  $600\Omega$  is connected to the sending end. Calculate the voltage and current at the receiving end. (4 marks)

QUESTION 3

- (a) A transmission line, 15km long is terminated in its characteristic impedance of  $600\angle 0^\circ \Omega$ . The sending end voltage is  $5\angle 0^\circ V$  at an angular frequency of  $10^4$  rad/sec and the resulting voltage at the receiving end is measured as  $0.77\angle -80^\circ V$ . Calculate the primary constants R, L, G and C of the line per kilometre. (8 marks)
- (b) A loss free line of length  $l$  of characteristic impedance  $Z_o$  (purely resistive) is terminated by an impedance  $Z_L = Z_o e^{j\theta}$ . Show that the magnitude of the input impedance can be written as:

$$Z_{in} = Z_o \left[ \frac{1 + \sin(2\beta l) \sin(\theta)}{1 - \sin(2\beta l) \sin(\theta)} \right]^{1/2} \quad (6 \text{ marks})$$

QUESTION 4

- (a) The smith chart is a graphical plot of the *normalised input impedance* of a transmission line, as well as its *complex reflection coefficient*, on a complex reflection coefficient plane. Denoting the normalised input impedance as  $\bar{z} = \bar{r} + j\bar{x}$ , and the complex reflection coefficient  $k = u + jv$ ,

- (i) Show that contours for constant normalised input resistance are given by

$$\left( u - \frac{\bar{r}}{1+\bar{r}} \right)^2 + v^2 = \frac{1}{(1+\bar{r})^2} \quad (5 \text{ marks})$$

- (ii) Show that contours for constant normalised input reactance are given by

$$(u-1)^2 + \left( v - \frac{1}{x} \right)^2 = \frac{1}{(x)^2} \quad (5 \text{ marks})$$

- (b) Show that for a loss free quarter wavelength line terminated by a load impedance  $Z_L$ , the input impedance is given by

$$Z_{in} = \frac{Z_o^2}{Z_L}. \text{ Deduce the behaviour of the line when it is open and short circuited.}$$

(4 marks)

QUESTION 5

- (a) Define the terms
- Voltage standing wave ratio
  - Reflection coefficient (4 marks)
- (b) A transmission line of negligible loss has a characteristic impedance of  $600\Omega$ . Calculate the VSWR along the line when the line is terminated in an impedance of  $(400+j300)\Omega$  (6 marks)
- (c) Two transmission lines of characteristic impedances  $Z_{o1}$  and  $Z_{o2}$  are joined together and a resistance  $R_1$  is placed across the junction. A wave of voltage  $V_1$  propagates towards the junction from the  $Z_{o1}$  side of the resistance. Find the expression for the reflection coefficient at the junction (4 marks)

QUESTION 6 ✓

- (a) Fig Qn 6 shows a loss free transmission line. Use the smith chart to determine
- (i) Normalised admittance of the load ✓
  - (ii) Reflection coefficient ✓
  - (iii) Voltage standing wave ratio ✓
  - (iv) Input impedance to the line ✓
- (6 marks)

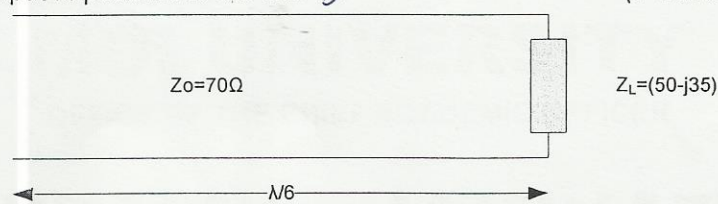


Fig Qn 6

- (b) Show that for a distortionless line, the propagation constant is given by

$$\gamma = j\omega\sqrt{LC} + R\sqrt{\frac{C}{L}} \quad (4 \text{ marks})$$

- (c) A loss free line of length  $l$  is terminated with a resistance equal to half its characteristic impedance. Show that the magnitude of the input impedance to the line is:

$$Z_{in} = Z_o \left[ \frac{1 + 4 \tan^2(\beta l)}{4 + \tan^2(\beta l)} \right]^{1/2} \quad (4 \text{ marks})$$

QUESTION 7 ✓

- (a) State two reasons why it is necessary to match the transmission line both to the load and the generator (2 marks)
- (b) A  $50\Omega$  transmission line is terminated with a load that is equal to  $Z_L = 75 + j65$ . Match the load to the transmission using a parallel open circuited stub. Determine in cm, the distance from the load to the point to insert the stub and the length of the stub when the frequency of operation is 1.5GHz. (Locate the stub as close as possible to the load) (5 marks)
- (c) A transmission line of characteristic impedance  $Z_o$  is terminated with a load impedance  $Z_L = R_L + jX_L$ . The load is matched to the line using lumped elements as shown in the fig Qn7. If  $X$  is the series reactance and  $B$  the shunt susceptance, show that for a matched condition

$$X = \pm \sqrt{R_L(Z_o - R_L)} - X_L$$

$$B = \pm \frac{\sqrt{(Z_o - R_L)/R_L}}{Z_o}$$

(7 marks)

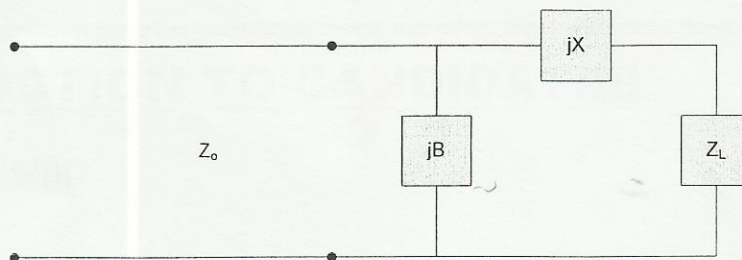


Fig Qn 7



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OFFICE OF THE CHIEF ACADEMIC OFFICER

## UNIVERSITY EXAMINATIONS

### 2011/2012 ACADEMIC YEAR

#### THIRD YEAR SECOND SEMESTER EXAMINATION

#### FOR THE DEGREE OF

### BACHELOR OF ENGINEERING

### IN

### ELECTRICAL AND COMMUNICATIONS

### ENGINEERING

**COURSE CODE:** ECE 342

**COURSE TITLE:** TRANSMISSION LINES

**DATE:** 29<sup>TH</sup> MAY, 2012 **TIME:** 9.00 A.M. - 12.00 NOON

### INSTRUCTION TO CANDIDATES

- ANSWER QUESTION ANY FIVE
- ALL QUESTIONS CARRY EQUAL MARKS.
- USE ONE SMITH CHART FOR ONLY ONE QUESTION
- THE FOLLOWING CONSTANTS ARE GIVEN

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ F/m}; \mu_0 = 4\pi \times 10^{-7} \text{ H/m} \quad c = 3 \times 10^8 \text{ m/s}$$

THIS PAPER CONSISTS OF (4) PRINTED PAGES

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✓ QUESTION 1

- (a) Using suitable diagrams, describe THREE types of transmission lines that are commonly in use. (3 marks)
- (b) Explain the following terms that are used to describe transmission lines
- (i) Uniform
  - (ii) Loss-less
  - (iii) Lossy
  - (iv) Distortionless (4 marks)
- (c) Draw an equivalent circuit of a section of transmission line of length  $\Delta z$  which has distributed parameters  $R\Omega/m$ ,  $LH/m$ ,  $GS/m$  and  $CF/m$  (3 marks)
- (d) Apply Kirchoff's voltage and current laws to the equivalent circuit, and show that for time harmonic signals

$$\frac{\partial^2 V}{\partial z^2} - (R + j\omega L)(G + j\omega C)V = 0 \quad (4 \text{ marks})$$

✓ QUESTION 2

- (a) A coaxial cable has the following distributed parameters per loop per kilometre at a frequency of 4MHz.  $R=110\Omega$ ,  $L=0.255mH$ ,  $C=0.07\mu F$  and  $G=0.003S$ . Calculate its:
- (i) attenuation constant
  - (ii) phase constant
  - (iii) characteristic impedance

The cable is to be used with intermediate repeaters in a telephone system. If the attenuation at 4MHz is not to exceed 50dB, calculate the maximum repeater spacing. (10 Marks)

- (b) Explain how a quarter wavelength lines can be used at high frequencies as tuned circuits in place of tuned resonant LC circuits? Under what conditions do the lines become capacitive and inductive? (4 marks)

QUESTION 3

- (a) Define the following terms
- ✓ (i) Reflection coefficient ✓
  - ✓ (ii) Voltage standing wave ratio ✓ (4 marks)
- (b) A lossless transmission line has a characteristic impedance of  $50\Omega$  and is terminated to a load of  $20+j50\Omega$ . Use the smith chart to determine:
- ✓ (i) the reflection coefficient
  - ✓ (ii) The VSWR on the line ✓
  - ✓ (iii) The impedance at a distance  $0.3\lambda$  from the load. ✓
  - ✓ (iv) The shortest distance from the load to a point on the line where the impedance is purely resistive and the value of this resistance (10 marks)

0.133  
0.

$$VSWR = \frac{V_{MAX}}{V_{MIN}} = \frac{V_1^0}{V_2^0}$$

0.457  
0.2 + j0.45

QUESTION 4

A loss free line of length  $l$  of characteristic impedance  $Z_0$  (purely resistive) is terminated by an impedance  $Z_L = Z_0 e^{j\theta}$ . Show that the magnitude of the input impedance can be written as:

$$Z_{in} = Z_0 \left( \frac{1 + \sin(2\beta l) \sin(\theta)}{1 - \sin(2\beta l) \sin(\theta)} \right)^{1/2} \quad (6 \text{ marks})$$

A 50Ω slotted line with a standing wave detector is used to measure the impedance of a load which is connected to the slotted line by an unknown length of 50Ω loss free coaxial cable. With the load in position, the VSWR was measured to be 2.2 and the adjacent voltage minima occur at the detector scale reading of 24.73cm and 87.23cm (scale readings increase in the direction of the load). When the load was replaced by a short circuit, the voltage minima moved to 9.1cm and 71.6cm. Use the smith chart to determine the load impedance.

⊖ V<sub>max</sub>

$$\frac{Z_L - Z_0}{Z_L + Z_0} = \frac{20 + j50 - 50}{20 + j50 + 50} \quad (8 \text{ marks})$$

$\frac{1}{2} \angle -30^\circ \times \frac{50}{70 + j50}$   
 $= 58.3 \angle -0.54^\circ$   
 $86.02 \angle 0.62^\circ$   
 $0.68 \angle -1.16^\circ$   
 $\rightarrow 0.27 - j0.6$

QUESTION 5

The smith chart is a graphical plot of the *normalised input impedance* of a transmission line, as well as its *complex reflection coefficient*, on a complex reflection coefficient plane. Denoting the normalised input impedance as  $\bar{z} = \bar{r} + j\bar{x}$ , and the complex reflection coefficient  $k = u + jv$ ,

(i) Show that contours for constant normalised input resistance are given by

$$\left( u - \frac{\bar{r}}{1 + \bar{r}} \right)^2 + v^2 = \frac{1}{(1 + \bar{r})^2} \quad (5 \text{ marks})$$

(ii) Show that contours for constant normalised input reactance are given by

$$(u - 1)^2 + \left( v - \frac{1}{\bar{x}} \right)^2 = \frac{1}{(\bar{x})^2} \quad (5 \text{ marks})$$

(b) Show that for a distortionless line, the propagation constant is given by

$$\gamma = j\omega\sqrt{LC} + R\sqrt{\frac{C}{L}} \quad (4 \text{ marks})$$

QUESTION 6

(a) Explain, giving reasons, why it is necessary to match the transmission line to both the load and the generator (4 marks)

(b) A transmission line of characteristic impedance 50Ω is used to connect a load of admittance  $Y_L = 0.036 + j0.030S$ . Design a double stub tuner to match this load to the line when the frequency is 500MHz (The stubs are short circuited and placed three eight wavelengths apart) (10 marks)

✓ QUESTION 7

Figure Qn 7 shows a coaxial line of internal and external conductor of radii a and b.

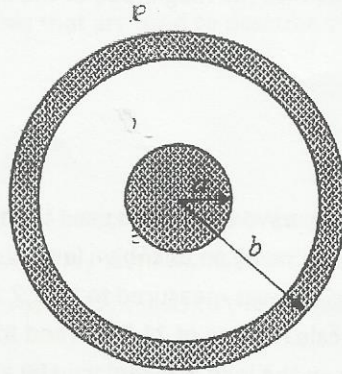


Fig Qn 7

- ✳ (a) Sketch the electric and the magnetic fields associated with the line. (2 marks)
- ✓ (b) Determine the expressions for the capacitance per unit length and the inductance per unit length of the line (8 marks) ✓
- ✓ (c) Assuming that the line is lossless determine the expression for the characteristic impedance of the line. (2 marks) (R)
- ✓ (d) A coaxial line of inner diameter of 0.29cm and sheath diameter of 1cm. Calculate its characteristic impedance of the cable if the relative dielectric permittivity is 2.3 (2 marks)

$$Z = \sqrt{C/L} = \frac{\frac{2\pi\epsilon}{\ln(b/a)}}{\frac{4\ln(b/a)}{2\pi}} = \frac{2\pi\epsilon}{\ln(b/a)} \times \frac{2\pi}{4\ln(b/a)} = \frac{4\pi^2\epsilon}{4\ln^2(b/a)} = \frac{\pi^2\epsilon}{\ln^2(b/a)}$$

$$Z = \frac{\ln(b/a)}{\sqrt{4\pi^2\epsilon}} = \frac{\ln(b/a)}{2\pi\sqrt{\epsilon}}$$



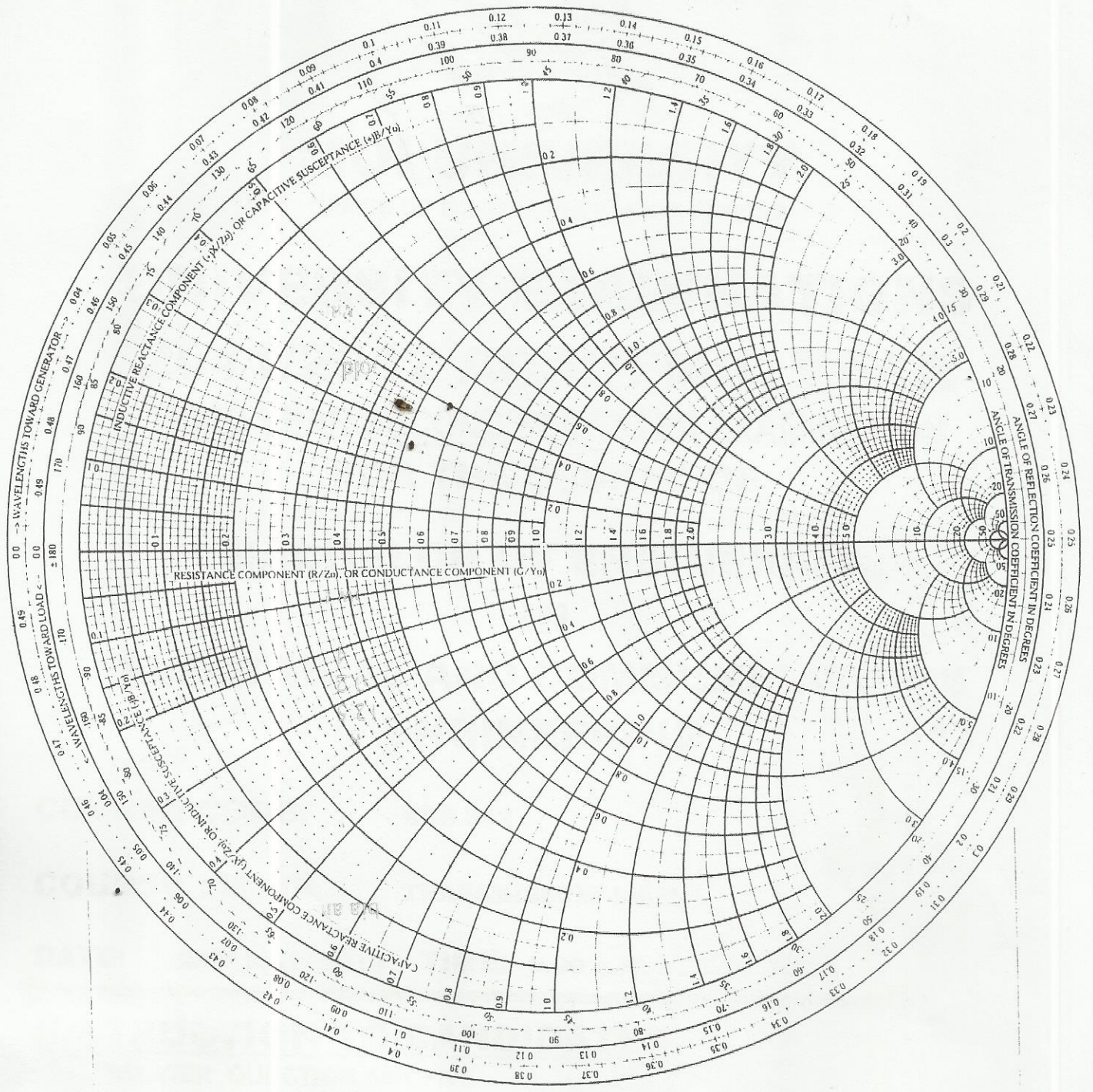
$$\epsilon_0 = 8.854 \times 10^{-12} \quad \mu_0 = 4\pi \times 10^{-7}$$

$$\frac{\ln(b/a)}{\sqrt{4\pi^2 \times 8.854 \times 10^{-12} \times \epsilon_r}}$$

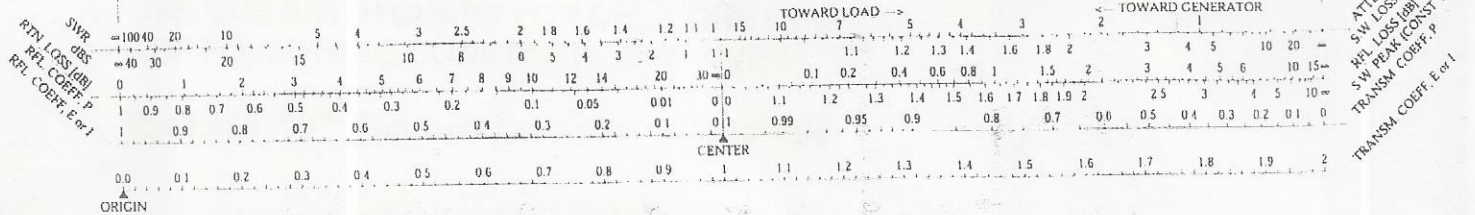


# The Complete Smith Chart

## Black Magic Design



### RADIALLY SCALED PARAMETERS



ATTEN (dB)  
SW LOSS COEFF  
REFL LOSS (dB)  
SW PEAK COEFF  
TRANSM COEFF P  
TRANSM COEFF E or I

$(0.5 - d) \times = 87$

$Z = 1.00$



- Explain the following terms used to describe transmission lines
  - Lossless
  - Lossy
  - Uniform
  - Distortionless
  - Reflection coefficient
  - Standing wave ratio (6 marks)

- A coaxial cable has the following distributed parameters per loop per kilometre at a frequency of 4MHz.  $R=110\Omega$ ,  $L=0.255mH$ ,  $C=0.07\mu F$  and  $G=0.003S$ . Calculate its:
  - attenuation constant
  - phase constant
  - characteristic impedance

$$\alpha = \frac{R}{2Z_0} = \frac{110}{2 \times 117} = 0.473$$

$$\beta = \omega \sqrt{LC} = 2\pi \times 4 \times 10^6 \times \sqrt{0.255 \times 10^{-3} \times 0.07 \times 10^{-6}} = 9.1009$$

$$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{0.255 \times 10^{-3}}{0.07 \times 10^{-6}}} = 117 \Omega$$

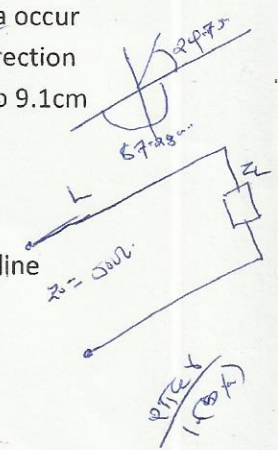
The cable is to be used with intermediate repeaters in a telephone system. If the attenuation at 4MHz is not to exceed 50dB, calculate the maximum repeater spacing (10 marks)

- A  $50\Omega$  slotted line with a standing wave detector is used to measure the impedance of a load which is connected to the slotted line by an unknown length of  $50\Omega$  loss free coaxial cable. With the load in position, the VSWR was measured to be 2.2 and the adjacent voltage minima occur at the detector scale reading of 24.73cm and 87.23cm (scale readings increase in the direction of the load). When the load was replaced by a short circuit, the voltage minima moved to 9.1cm and 71.6cm. Use the smith chart to determine the load impedance. (8 marks)

$$S = 2.2$$

$$S_{min} = 9.1cm$$

$$S_{max} = 71.6cm$$



- A transmission line of characteristic impedance  $50\Omega$  is used to connect a load of admittance  $Y_L = 0.036 + j0.030S$ . Design a double stub tuner to match this load to the line when the frequency is 500MHz (The stubs are open circuited and placed three eighth wavelengths apart) (8 marks)

- Determine the expression for the capacitance per unit length and the inductance per unit length for a coaxial line of internal and external radii a and b respectively (8 marks)



$$D = \epsilon E = \frac{Q}{2\pi r}$$

$$V = \int_a^b \frac{Q}{2\pi \epsilon r} dr = \frac{Q}{2\pi \epsilon} \ln \frac{b}{a}$$

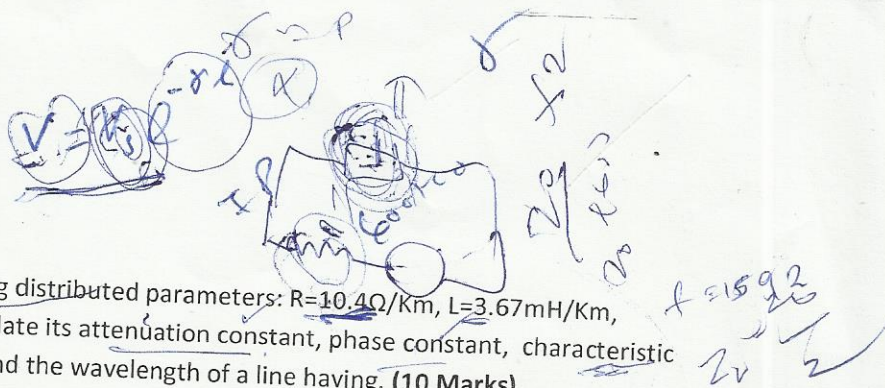
$$C = \frac{Q}{V} = \frac{2\pi \epsilon \ln \frac{b}{a}}{1} \text{ p/m}$$

$$\Phi = \int_a^b \frac{Q}{2\pi r} dr = \frac{Q}{2\pi} \ln \frac{b}{a}$$

$$L = \frac{\Phi}{I} = \frac{Q}{I} \frac{1}{2\pi} \ln \frac{b}{a} = \frac{\mu \ln \frac{b}{a}}{2\pi} \text{ p/m}$$

ECE 342 – TRANSMISSION LINES

CAT 1 – 7<sup>th</sup> March 2011



Q1. A 300m long line has the following distributed parameters:  $R=10.4\Omega/\text{Km}$ ,  $L=3.67\text{mH}/\text{Km}$ ,  $C=8.35\text{pF}/\text{Km}$  and  $G=0.8\mu\text{S}/\text{Km}$ . Calculate its attenuation constant, phase constant, characteristic impedance, velocity of propagation and the wavelength of a line having. (10 Marks)

The line is terminated at by its characteristic impedance, and a 2V generator of internal impedance  $600\Omega$  is connected to the sending end. Calculate the voltage and current at the receiving end. (5 marks)

Q2. A loss free line of length  $l$  is terminated with a resistance equal to half its characteristic impedance,  $Z_0$ . Show that the magnitude of the input impedance to the line is

$$Z_{in} = Z_0 \left[ \frac{1 + 4 \tan^2(\beta l)}{4 + \tan^2(\beta l)} \right]^{1/2} \quad (5 \text{ marks})$$

Handwritten derivation for Q2:

$$Z_{in} = Z_0^2 \frac{\cos^2 \beta l + 2 \sin^2 \beta l}{\sin^2 \beta l + 2 \cos^2 \beta l}$$

Since  $\beta l = jBL$ ,  $3 + \frac{6}{8}$  is also written.

Q3. A loss free transmission line of characteristic impedance  $Z_0=70\Omega$ , and length  $l=\frac{\lambda}{6}$  is terminated by a load impedance of  $Z_L=50-j35\Omega$ . Using the smith chart, obtain the input impedance to the line. Determine also the impedance, when the line is open and short circuited (10 Marks)