EGERTON UNIVERSITY

UNIVERSITY EXAMINATIONS

NJORO CAMPUS

SECOND SEMESTER, 2013/2014 ACADEMIC YEAR

SECOND YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN ENGINEERING

MATH 215: ENGINEERING MATHEMATICS

STREAM: BSC Y3S2 AGEN, WEEN, ICEN **TIME:** 2 HOURS

DAY: MONDAY, 8.30 – 11.30 AM **DATE:** 19/05/2014

INSTRUCTIONS:

- **Answer question one and any two other questions**
- **A new question must be started on a new page**

QUESTION ONE (30 MARKS)

(a) Show that $\frac{1+2i}{3-4i}+\frac{2-i}{5i}$ reduces to a real number. (5 marks)

(b) Write $z=\frac{-2}{1+i\sqrt{3}}$ in exponential form. (5 marks)

(c) Find the equation of a straight line passing through the points (-4,2,7) and (2,3,10) (3 marks)

(d) In the following matrix, find its determinant and hence its inverse.

$$A=\begin{pmatrix} 3 & 2 & -1 \\ 2 & -5 & 2 \\ 5 & 16 & -7 \end{pmatrix}$$ (4 marks)

(e) Solve the differential equation

$$2\sqrt{x}\frac{dy}{dx}=x^2-1$$ (4 marks)

(f) Suppose a bacterial culture initially has 400 cells. After one hour, the population has increased to 800. Find an equation for the population at any time. What will be the population be after 10 hours. (5 marks)

(g) Find a unit vector that is perpendicular to both the vector

$\hat{a} = <4, 1, -3>$ *and* $\hat{b} = <-1, -2, 5>$ (4 marks)

QUESTION TWO (20 MARKS)

(a) A triangle has vertices at A(6, 11, 3), B(1, 5, -7) and C(2, 0, 2). Find the angles at each vertex. (5 marks)

(b) If $\hat{a} = <1, t, 1>$ and $\hat{b} = <9, 1, 1>$ find the value of t such that $Proj_{\hat{b}}\ \hat{a} = \hat{b}$ (5 marks)

(c) Find the equation of a plane containing the points (0, 6, 0), (-5, 7, 1) and (2, 1, 2). (5 marks)

(d) Find the area of a parallelogram spanned by the vectors $\hat{a} = <4, 2, -1>$ and $\hat{b} = <-1, 10, 7>$. (5 marks)

QUESTION THREE (20 MARKS)

(a) Write the complex number $z = (\sqrt{3} - i)^6$ in std form. (5 marks)

(b) Solve the equation $Z^2 - 1 = 0$ for $0 \leq \theta \leq 2\pi$. (5 marks)

(c) Given that z = 1 + i, show that z^{10} is a purely imaginary number. (5 marks)

(d) Use de moivre's theorem to express $\sin 4\theta$ and $\cos 4\theta$ in terms of powers of $\sin\theta$ and $\cos\theta$. (5 marks)

QUESTION FOUR (20 MARKS)

(a) Solve the following system of equations by Cramer's rule

$$x - y - z = 0$$
$$3x + y + 2z = 6$$
$$2x + 2y + z = 2$$

(6 marks)

(b) Two matrices are given as

$$A = \begin{bmatrix} 1 & -1 & 1 \\ -3 & 2 & -1 \\ -2 & 1 & 0 \end{bmatrix} \quad and \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 2 & 3 \end{bmatrix}$$

Find AB and BA. (6 marks)

(c) Find the inverse of the matrix A given by

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{pmatrix}$$

and show that $AA^{-1} = I$ (8 marks)

QUESTION FIVE (20 MARKS)

(a) Solve the following differential equation by separation variables.

$xy' = 1 - x^2$ (5 marks)

(b) Find the solution of the following homogeneous equation

$(x^2 + y^2)dx - 2xydy = 0$ (6 marks)

(c) Show that the differential equation is exact and find its general solution

$y^2dx + (2xy + \cos y)dy = 0$ (4 marks)

(d) Scientist dating a fossil estimate that 20% of the original amount of Carbon-14 is present. If the half life is 5730 years, approximately how old is the fossil? (5 marks)

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