

EGERTON



UNIVERSITY

UNIVERSITY EXAMINATIONS  
NJORO CAMPUS

FIRST SEMESTER, 2013/2014 ACADEMIC YEAR

SECOND YEAR EXAMINATION FOR THE DEGREES BACHELOR OF SCIENCE,  
ARTS & EDUCATION  
MATH 211: CALCULUS II

STREAM: Y2S1 (BA, B.ED, BSC)

TIME: 2 HRS

DAY: THURSDAY 8.30 – 11.30 A.M.

DATE: 09/1/2014

## INSTRUCTIONS:

*Answer Question ONE and any other TWO Questions*

Question One – 30 Marks

(a) Evaluate the following integrals:

(i)  $\int_{-1}^1 (x+1)(x^2+2x)^4 dx$  (3 Marks)

(ii)  $\int x \sec^2 x dx$  (3 Marks)

(iii)  $\int \frac{3x+1}{x^3+2x^2} dx$  (4 Marks)

$x(x+1) + 1(x+1)$   
 $= x^2 + x + x + 1$   
 $= x^2 + 2x + 1$   
 $= (x+1)^2$   
 $u = x+1$

(b) Evaluate the improper integral:  $\int_3^{\infty} \frac{1}{(x-2)^3} dx$  (3 Marks)

(c) Find all values of  $x$  satisfying the Lagrange's mean value theorem for the function  $f(x) = 4 + 3x - 2x^2$  over the interval  $[1,3]$ . (4 Marks)

\* (c) Use the Trapezoidal rule with  $n=8$  to estimate  $\int_2^3 \frac{1}{\sqrt{x+2}} dx$ . (5 Marks)

h ?

(d) The region bounded by the curve  $y = x^2$  and the lines  $x = -2$  and  $x = 2$  is revolved about the line  $y = -1$ . Calculate the volume generated. (5 Marks)

(g) Given that  $f(x, y) = \sin(2x + 3y)$ , find  $\frac{\partial f}{\partial x}$  and  $\frac{\partial f}{\partial y}$  (3 Marks)

**Question Two – 20 Marks**

(a) Expand  $f(x) = \frac{1}{3x+2}$  in Taylor's series at  $x = 2$  up to the term involving  $x^3$ . (5 Marks)

(b) Verify Rolle's theorem for the function  $f(x) = x^2 - 3x + 2$  over the interval  $[1,2]$  (4 Marks)

$x^2 - 3x + 2$

(c) Decompose the function  $f(x) = \frac{x}{16x^4 - 1}$  into partial fractions and hence evaluate

$\int \frac{x}{16x^4 - 1} dx$  (6 Marks)

(d) Calculate the length of the arc of the curve  $y = \frac{x^5}{10} + \frac{1}{6x^3}$  from  $x = 1$  to  $x = 2$  (5 Marks)

**Question Three – 20 Marks**

Evaluate the following integrals:

(a)  $\int_0^9 \frac{x^2 + 4}{x(x-1)^2} dx$   $\frac{x^2+4}{x} + \frac{x^2+4}{(x-1)^2}$  (6 Marks)

(b)  $\int \frac{dx}{x^2 \sqrt{x^2 + 1}}$  (5 Marks)

(c)  $\int x \tan^{-1} x dx$  (5 Marks)

(d)  $\int_0^9 \frac{dx}{1 + \cos x}$  (4 Marks)

**Question Four – 20 Marks**

(a) Evaluate the following improper integral:  $\int_0^3 \frac{dx}{\sqrt{9-x^2}}$  let  $u = 9-x^2$   
 $du = -2x dx$  (5 Marks)

*Handwritten notes:* It isn't continuous at  $x=3$ .  
Add  $\lim_{h \rightarrow 0} \int_0^{3-h} \frac{dx}{\sqrt{9-x^2}}$

(b) Use the Simpson's rule rule with  $n=6$  to estimate  $\int_0^\pi \frac{\sin x}{x} dx$ . (5 Marks)

$A = h/3 [y_0 + 2(\text{even}) + 4(\text{odd}) + y_n]$   $h = \frac{b-a}{n}$

(c) Find the area of the region enclosed by the line  $y = x - 1$  and the parabola  $y^2 = 2x + 6$  (5 Marks)

(d) Find the volume generated when the area enclosed by the curves  $y = x^2$  and  $y = 4x - x^2$  is revolved about the line  $y=6$ . (5 Marks)

*Handwritten notes:* let  $x = 9 \sin u$   
 $x = 3 \sin u$

**Question Five – 20 Marks**

- (a) Find the first partial derivatives of the function  $z = \tan^{-1} xy^2 + \tan^{-1} x^2 y$ . (4 Marks)

$$\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$$

- (b) Evaluate the following iterated integral  $\iint_R 6xy dA$  where  $R$  is the region in the first quadrant bounded by the graphs  $y = 2x$  and  $y = x^2$ . (5 Marks)

- (c) Find the length of the arc of the catenary  $y = \frac{1}{2}a \left( e^{\frac{x}{a}} + e^{-\frac{x}{a}} \right)$  from  $x=0$  to  $x=a$ .

$$s = \int_0^a \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

(5 Marks)

- (d) Evaluate:  $\int_{-1}^{1+x^2} \int (3x + 2y) dy dx$  (4 Marks)