EGERTON UNIVERSITY

UNIVERSITY EXAMINATIONS

NJORO CAMPUS

2012/2013 ACADEMIC YEAR

THIRD YEAR EXAMINATION FOR THE AWARD OF THE DEGREE OF BACHELOR SCIENCE

MATH 318: CALCULUS III

STREAM: B.Sc., B.ED (SCI) **TIME:** 2 HOURS

DAY: FRIDAY, 12.00 – 2.00 P.M. **DATE:** 18/01/2013

INSTRUCTIONS:

(i) Answer question ONE and ANY OTHER TWO questions.
(ii) Write neat and well organised solutions

QUESTION ONE: 30 MARKS

a) Find the partial derivatives of the following functions:

(i) $f(x,y)=\dfrac{y\sqrt{y}}{1-x}$ (2 marks)

(ii) $w=\ln(xy+yz)$ (3 marks)

b) Evaluate the following:

$$\int_0^2\int_3^y (y-x)\,dxdy$$ (4 marks)

c) Calculate the total differential of $f(x,y,z)=x^2\cos yz$ (3 marks)

d) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ of the equation $x^2 - 2y^2 + 3z^2 + yz - xy + 2 = 0$ (4 marks)

e) Write the general term of the series below:

$\frac{9}{5} + \frac{25}{8} + \frac{49}{11} + \frac{81}{14} + \ldots\ldots\ldots\ldots\ldots$ (3 marks)

f) Determine the critical points of the surface:

$f(x, y) = x^3 - 3x + y^3 - 3y$ (4 marks)

g) State with reasons, whether the following series converge or diverge:

(i) $\sum_{n=1}^{\infty} \frac{3n^2}{2n^2 - 1}$

(ii) $\sum_{n=1}^{\infty} \frac{1}{5n}$

(4 marks)

h) Determine the value of:

$\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^{2n}$ (3 marks)

QUESTION TWO: 20 MARKS

a) Evaluate the following iterated integrals:

(i) $\int_0^1\int_1^2 \frac{1}{\sqrt{1-x^2}}\,dydx$ (4 marks)

(ii) $\int_0^2\int_0^y \frac{2}{y^2+1}\,dxdy$ (4 marks)

b) Evaluate $\iint_R f(x,y)\,dA$ where R is the region bonded by

$y=x^2$ and $y=4$ and $f(x,y)=2x^2y.$ (6 marks)

c) Find the volume of solid between the surface

$f(x,y)=\cos(x+2y)$ and the domain bounded by $x=y, x=3y, y=0$ and $y=\pi$

(6 marks)

QUESTION THREE: 20 MARKS

a) Write down the equation of the tangent plane and normal line to the surface

$f(x,y)=x^3-3x^2y^3+6$ at the point where $x=-1$ and $y=-1$ (6 marks)

b) Determine and classify all the critical points of the function:

$f(x,y)=y^3+3yx^2-3y^2-3x^2+4.$ (8 marks)

c) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ given that $z=2uv$ and $u=xy$ and $v=\frac{x}{y}$ (6 marks)

QUESTION FOUR: 20 MARKS

a) Express the decimal number 22.2222..........as a rational number. (4 marks)

b) Consider the series:

$$\frac{9}{1!}+\frac{9^2}{2!}+\frac{9^3}{3!}+\frac{9^4}{4!}+........+\frac{9^n}{n!}+.......$$

Find $r=\lim_{n\to\infty}\frac{a_{n+1}}{a_n}$ and state whether the series is convergent or divergent.

(5 marks)

c) Find the sum of the infinite series $\sum_{n=1}^{\infty}\frac{1}{(n+1)(n+2)}$ (use partial sums) (6 marks)

d) Find the sum of the infinite series $\sum_{n=1}^{\infty}\frac{1}{3^n}$ (5 marks)

QUESTION FIVE: 20 MARKS

a) By method of differentials find value of $\left(\sqrt{17}+\sqrt{101}\right)^2$ (5 marks)

b) Find the function $f(x,y)$ satisfying the following conditions.

$f_x=6x+12,$ $\quad f_y=9y^2-8y,$ $\quad f(-1,2)=10.$ (5 marks)

c) Find $\frac{df}{dx}$ given that $f(u,v)=u^2+e^{2v}$ where $u=e^x$ and $v=\ln x$ (5 marks)

d) Maximize $f(x,y)=x^2+y^2$ subject to $2x-y=4.$ (5 marks)
