MATH 318. 0.9

MATH 318

Problem Set 2

Find the first partial derivatives of the function

(a) $f(x, y) = \sqrt{4 - x^2 - 9y^2}$

(b) $f(x, y z) = x^2 y^5 + x z^2$

(c) $z = (\sin x^2 y)^3$

(d) $f(x, y, z) = \dfrac{x + y + z}{xy + yz + zx}$.

(e) $w = e^x (\cos y + \sin z)$

(f) $w = \left(\dfrac{x}{y}\right)^z$.

2 Find $f(xy)$ and $f(yx)$

(a) $f(x, y) = 3x^2 - \sqrt{2}\, x y^2 + y^5 - 2$

(b) $f(x, y) = \dfrac{x^2 - y^2}{x^2 + y^2}$

(c) $f(x, y, z) = z(\cos xy)$

Satisfies the Cauchy - Reimann's eqns.

4) Show that $z = x^4 - 6x^2y^2 + y^4$ satisfies the equation

$$\frac{\delta^2 z}{\delta x^2} + \frac{\delta^2 z}{\delta y^2} = 0.$$

5. Let m_1 and m_2 be masses with $m_1 \geq m_2$, and assume that they are connected by an ~~pulley~~ apparatus called Atwood machine. The accelaration of the mass m_1 downward is given by

$$a = \frac{m_1 - m_2}{m_1 + m_2} g$$ where g

is the ~~gravitational~~ ~~force~~ accelaration due to gravity: show that

$$m_1 \frac{\delta a}{\delta m_1} + m_2 \frac{\delta a}{\delta m_2} = 0.$$

(6) When two resistors having resistances R_1 and R_2 are connected in parallel the combined resistance R is given by.

Show That

$$\left(\frac{\delta^2 R}{\delta R_1^2}\right)\left(\frac{\delta^2 R}{\delta R_2^2}\right) = \frac{4R^2}{(R_1+R_2)^4}$$

) For the gas formula $\left(P+\frac{a}{V^2}\right)(V-b)=ct$ where a b and c are constants show that

$$\frac{\partial P}{\partial V} = \frac{2a(V-b)-\left(P+a/V^2\right)V^3}{V^3(V-b)}$$

$$\frac{\partial V}{\partial t} = \frac{cV^3}{\left(P+a/V^2\right)V^3-2a(V-b)}$$

$$\frac{\delta t}{\delta P} = \frac{V-b}{c},$$

Show that

$$\left(\frac{\delta P}{\delta V}\right)\left(\frac{\delta V}{\delta t}\right)\left(\frac{\delta t}{\delta P}\right) = -1 \Rightarrow \text{cyclic relation.}$$

ercises 12.3

lating First Order Partial Derivatives

ercises 1–22, find $\partial f/\partial x$ and $\partial f/\partial y$.

$f(x, y) = 2x^2 - 3y - 4$ 2. $f(x, y) = x^2 - xy + y^2$

$f(x, y) = (x^2 - 1)(y + 2)$

$f(x, y) = 5xy - 7x^2 - y^2 + 3x - 6y + 2$

$f(x, y) = (xy - 1)^2$ 6. $f(x, y) = (2x - 3y)^3$

$f(x, y) = \sqrt{x^2 + y^2}$ 8. $f(x, y) = (x^3 + (y/2))^{2/3}$

$f(x, y) = 1/(x + y)$ 10. $f(x, y) = x/(x^2 + y^2)$

$f(x, y) = (x + y)/(xy - 1)$

$f(x, y) = \tan^{-1}(y/x)$ 13. $f(x, y) = e^{(x+y+1)}$

$f(x, y) = e^{-x} \sin(x + y)$ 15. $f(x, y) = \ln(x + y)$

$f(x, y) = e^{xy} \ln y$ 17. $f(x, y) = \sin^2(x - 3y)$

$f(x, y) = \cos^2(3x - y^2)$ 19. $f(x, y) = x^y$

$f(x, y) = \log_y x$

$f(x, y) = \int_x^y g(t)\,dt$ (g continuous for all t)

$f(x, y) = \sum_{n=0}^{\infty} (xy)^n$ ($|xy| < 1$)

Exercises 23–34, find f_x, f_y, and f_z.

$f(x, y, z) = 1 + xy^2 - 2z^2$ 24. $f(x, y, z) = xy + yz + xz$

$f(x, y, z) = x - \sqrt{y^2 + z^2}$

$f(x, y, z) = (x^2 + y^2 + z^2)^{-1/2}$

$f(x, y, z) = \sin^{-1}(xyz)$ 28. $f(x, y, z) = \sec^{-1}(x + yz)$

$f(x, y, z) = \ln(x + 2y + 3z)$

$f(x, y, z) = yz \ln(xy)$ 31. $f(x, y, z) = e^{-(x^2+y^2+z^2)}$

$f(x, y, z) = e^{-xyz}$

$f(x, y, z) = \tanh(x + 2y + 3z)$

$f(x, y, z) = \sinh(xy - z^2)$

xercises 35–40, find the partial derivative of the function with ct to each variable.

$f(t, \alpha) = \cos(2\pi t - \alpha)$ 36. $g(u, v) = v^2 e^{(2u/v)}$

$(\rho, \phi, \theta) = \rho \sin\phi \cos\theta$

$(r, \theta, z) = r(1 - \cos\theta) - z$

Vork done by the heart. (Section 3.7, Exercise 56)

$$W(P, V, \delta, v, g) = PV + \frac{V\delta v^2}{2g}$$

Vilson lot size formula. (Section 3.6, Exercise 57)

$$A(c, h, k, m, q) = \frac{km}{q} + cm + \frac{hq}{2}$$

Calculating Second Order Partial Derivatives

Find all the second order partial derivatives of the functions in Exercises 41–46.

41. $f(x, y) = x + y + xy$ 42. $f(x, y) = \sin xy$
43. $g(x, y) = x^2y + \cos y + y \sin x$
44. $h(x, y) = xe^y + y + 1$ 45. $r(x, y) = \ln(x + y)$
46. $s(x, y) = \tan^{-1}(y/x)$

Mixed Partial Derivatives

In Exercises 47–50, verify that $w_{xy} = w_{yx}$.

47. $w = \ln(2x + 3y)$ 48. $w = e^x + x \ln y + y \ln x$
49. $w = xy^2 + x^2y^3 + x^3y^4$ 50. $w = x \sin y + y \sin x + xy$
51. Which order of differentiation will calculate f_{xy} faster: x first, or y first? Try to answer without writing anything down.
   a) $f(x, y) = x \sin y + e^y$
   b) $f(x, y) = 1/x$
   c) $f(x, y) = y + (x/y)$
   d) $f(x, y) = y + x^2y + 4y^3 - \ln(y^2 + 1)$
   e) $f(x, y) = x^2 + 5xy + \sin x + 7e^x$
   f) $f(x, y) = x \ln xy$
52. The fifth order partial derivative $\partial^5 f/\partial x^2 \partial y^3$ is zero for each of the following functions. To show this as quickly as possible, which variable would you differentiate with respect to first: x, or y? Try to answer without writing anything down.
   a) $f(x, y) = y^2x^4e^x + 2$
   b) $f(x, y) = y^2 + y(\sin x - x^4)$
   c) $f(x, y) = x^2 + 5xy + \sin x + 7e^x$
   d) $f(x, y) = xe^{y^2/2}$

Using the Partial Derivative Definition

In Exercises 53 and 54, use the limit definition of partial derivative to compute the partial derivatives of the functions at the specified points.

53. $f(x, y) = 1 - x + y - 3x^2y$, $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$ at $(1, 2)$
54. $f(x, y) = 4 + 2x - 3y - xy^2$, $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$ at $(-2, 1)$
55. Let $w = f(x, y, z)$ be a function of three independent variables, and write the formal definition of the partial derivative $\partial f/\partial z$ at (x_0, y_0, z_0). Use this definition to find $\partial f/\partial z$ at $(1, 2, 3)$ for $f(x, y, z) = x^2yz^2$.
56. Let $w = f(x, y, z)$ be a function of three independent variables and write the formal definition of the partial derivative $\partial f/\partial y$ at (x_0, y_0, z_0). Use this definition to find $\partial f/\partial y$ at $(-1, 0, 3)$ for $f(x, y, z) = -2xy^2 + yz^2$.

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MATHEMATICS DEPT
MATH318:CALCULUSIII
C.A.T II

Q1 Show that the following improper integral converges and compute its value

$$\int_{-\infty}^{\infty} \frac{dx}{e^{x}+e^{-x}}$$

Q2 (a) Show that the function $f(x,y)=\dfrac{2x^2y}{x^4+y^2}$ has no limit as $(x,y)\to(0,0)$

(b) Show that the function $f(x,y,z)=\ln\sqrt{x^2+y^2+z^2}$ satisfies the Laplace Equation.

(c) Find $\dfrac{\partial z}{\partial x}$ *and* $\dfrac{\partial z}{\partial y}$ if $z^3-xy+yz+y^3=2$

Q3 Let $T=f(x,y)$ be the temperature at (x,y) on the circle $x=\cos t$ *and* $y=\sin t$ and suppose $\dfrac{\partial T}{\partial x}=8x-4y$ $\dfrac{\partial T}{\partial y}=8y-4x$

(a) Find where the maximum and minimum temperatures on the circle occurs by examining the derivatives $\dfrac{\partial T}{\partial t}$ *and* $\dfrac{\partial^2 T}{\partial t^2}$

(b) Suppose $T=4x^2-4xy+4y^2$ Find the maximum and Minimum values of t on the circle

Q4 Find the absolute maximum and minimum values of $f(x,y)=x^2-xy+y^2+1$ on the triangular plate in the first quadrant bounded by the lines $y=x, y=4$ *and* $x=0$

$(y-x)(z-y)$

$-(z-y)^{-1}$

~~0733437112~~

~~0728666185~~

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MATHEMATICS DEPARTMENT
MATH 318: C.A.T 1 30-10-07

Attempt All The Questions

1. Find the partial derivatives of $f(xy) = \dfrac{xy}{x^2y - xy^2}$

$$\frac{\partial f}{\partial x} = \frac{(xy)'(x^2y - xy^2) - [(x^2y - xy^2)'(xy)]}{(x^2y - xy^2)^2}$$

$$= \frac{(y^2x^2 - xy^3) - (2xy - y^2)xy}{x^2y(x^2y - xy^2) - xy^2(x^2y - xy^2)}$$

$$= \boxed{\frac{y^2x^2 - xy^3 - (2x^2y^2 - xy^3)}{x^4y^2 - x^3y^3 - x^3y^3 + x^2y^4}}$$

$$\frac{y^2x^2 - \cancel{xy^3} - 2x^2y^2 + \cancel{xy^3}}{x^2y^2(x^2 - xy - xy + y^2)}$$

$$= \frac{-y^2x^2}{x^2y^2(x^2 - xy - xy + y^2)}$$

$$= \frac{-1}{x^2 - 2xy + y^2}$$

2. Evaluate the following iterated integral $\displaystyle\int_1^2\int_0^{x^2} xy\,dy\,dx$

3. Determine and classify all the critical points of the function

$$f(x,y) = y^3 + 3x^2y - 12y$$

4. Find the equation of the tangent plane and normal line to the surface $z^2 = x^2 - y^2$ at the point $(5,-4,-3)$

5. Find the volume of the solid formed between the surface $f(x,y) = x^2 + y^2$ and above the domain bounded by $y = x^2$ and $y = 1$.

6. Find the total differential of the function $u = x + \dfrac{y-x}{z-y}$.

7. Evaluate the rational number equivalent to infinite decimal number 1.13232323232.............

Ht3

$f_x = 6xy \rightarrow 6xy = 0$

$f_y = 3y^2 + 3x^2 - 12 \quad y^2 + x^2 - 4 = 0$

$\frac{x}{2}$

pts $(0,2)$

$(0,-2)$

$(-2,0)$

$(2,0)$

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MATHEMATICS DEPARTMENT
MATH 318 : CALCULUS III

Attempt All Questions

Question One

~~a)~~ Define precisely the " Sequence $\{a_n\}$ converges to a limit L"

b) Find the limits of the following sequences whose general terms are given by

~~(i)~~ $a_n = \sqrt{\dfrac{2n}{n+1}}$

(ii) $a_n = \dfrac{(n+1)(n-1)}{2n^2}$

(iii) $a_n = \dfrac{1}{n}\left(\sqrt{n+2} - \sqrt{n}\right)$

Question Two

a) Express the recurring decimal 1.414141 ...... as a ratio of two integers.

~~b)~~ Find the n^{th} - partial sum of the series $\sum_{n=1}^{\infty} \dfrac{1}{(n+1)(n+2)}$ .Hence or otherwise find its sum.

(c)Determine whether or not the following series converges

(i) $\sum_{n=1}^{\infty} \dfrac{(-3)^n}{n!}$

~~(ii)~~ $\sum_{n=1}^{\infty} \left(\dfrac{1}{n^2\sqrt{n}}\right)$

* (iii) $\sum_{n=1}^{\infty} \dfrac{1}{n^2+3n}$ (Hint: use the integral test)

* **Question 3**

Find the interval of convergence of the power series

$$\sum \frac{(-1)^{n-1}(x-2)^n}{3^n n}.$$

$1 \times 2 \times 3 \times 4 \times 5 = 5!$

MATH318:CALCULUSIII

$(n+1)!$

PROBLEM SET1

$(n+1)(n!)$

(A)

Obtain the limit of sequence having the following as the nth term. Test the convergence of the sequences and determine limits when they are convergent

$n+1$

1) $x_n = \dfrac{2n^2+1}{2n^2+n}$ 2) $x_n = \dfrac{n+1}{n^3}$ 3) $x_n = \dfrac{n^3+1}{2n^2+n+5}$

4) $x_n = \sqrt{n+1} - \sqrt{n}$ 5) $x_n = \dfrac{(n+1)!}{n!}$ 6) $x_n = \dfrac{1}{3^{2n-1}}$

(B)

1) Find the series having $\{\ln(n+1)\}$ as its sequence of partial sums

2) Express the recurring decimal 1.22222.... as a ratio of integers

3) Find the sums of the following series

(a) $\sum_{n=1} \ln\left(\dfrac{n}{n+1}\right)$

(b) $\sum_{n=1}^{\infty} \dfrac{-2}{n(n+1)}$

(c) $\sum_{n=1}^{\infty} (-1)^n \dfrac{3}{4^n}$

4) Find the nth partial sum of the series $\sum_{n=1}^{\infty} \dfrac{3n-2}{n(n+1)(n+2)}$. Hence or otherwise find its limit

5) Determine whether or not the following series converge

(a) $\sum_{n=1}^{\infty} \dfrac{(n!)^2}{(2n)!}$

(b) $\sum_{n=1}^{\infty} \left(\dfrac{n}{n+1}\right)^{n^2}$

(c) $\sum_{n=1}^{\infty} \dfrac{(-3)^n}{n!}$

$5!$

$1 \times 2 \times 3 \times 4 \times 5$

$= n+1 \quad n+2 \cdot n+3 \quad n+1 = 5$

8.44
2.00
10.44

$u \cdot v$

$\frac{u}{v} = \frac{u'v - v'u}{v^2}$

EGERTON UNIVERSITY

UNIVERSITY EXAMINATIONS
NJORO CAMPUS

FIRST SEMESTER 2007/2008

THIRD YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE AND BACHELOR OF EDUCATION

MATHS 318: CALCULUS III

STREAM: BSc. BED **TIME:** 2 HRS

DAY: THURSDAY 8.30 – 11.30 A.M. **DATE:** 13/12/2007

SECTION A: (Answer Questions one and any other TWO questions

QUESTION ONE [30 Marks]

$f(x,y) = \left(\frac{x+1}{y}\right)^2$

$f_x = 2\left(\frac{x+1}{y}\right)\cdot\frac{1}{y} = \frac{2(x+1)}{y^2}$

$f_y = 2\left(\frac{x+1}{y}\right)\left(\frac{x+1}{y^2}\right) = \frac{2(x+1)^2}{y^3}$

a) Find the partial derivations of the following functions:

i) $f(x,y) = \left(\frac{x+1}{y}\right)^2$ [2 marks]

$\frac{\partial f}{\partial x} = 2\left(\frac{x+1}{y}\right)\cdot\frac{y}{y^2 y} = \frac{2(x+1)}{y^2}$

$f_y = \frac{x+z}{(xy+yz+xz)}$

ii) $f(x,y,z) = In(xy+yz+xz)$ [3 marks]

$f_x = \frac{y+z}{(xy+yz+xz)}$

$(x^2+1)^{-1}$

$f_z = \frac{y+x}{(xy+yz+xz)}$

b) Evaluate the following iterated integral

$$\int_0^1\int_0^2 \frac{1}{x^2+1}\,dy\,dx$$ [3 marks]

$f(x,y,z) = \ln(xyz)^{-1}$

$\frac{y}{x^2+1}\Big|_0^2 dx$

$(xyz)^{-1} = yz$ $(xyz)^{-1}$ $yz($

c) Calculate the total differential of the function

$$f(x,y,z) = Sin\left(\frac{1}{xyz}\right)$$

[3 marks]

$(xyz)^{-1}$

$3y^2 + y + 0$ $\frac{2}{16} + \frac{1}{25}$

d) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ given that $x^2 - 2y^2 + 3z^2 - yz - y = 0$ [3 marks]

$\frac{16}{100}$

e) Find the sum of the following infinite series

$$2 - 1 + \frac{2}{4} - \frac{1}{5} + \frac{2}{4^2} - \frac{1}{5^2} + \frac{2}{4^3} - \frac{1}{5^3} + \ldots\ldots\ldots\ldots$$

[4 marks]

$2 - 1 + \frac{2}{4}$ 0.16

f) Find the critical points of the surface given by

$$f(x,y) = 2x^2 - y^3 - 2xy$$

[4 marks]

g) Find the maximum and minimum value of the function

$f(x,y) = 4xy$ subject to $x^2 + y^2 = 4$

[4 marks]

$f_x = 4y$ $g_x = 2x$
$f_y = 4x$ $g_y = 2y$
$4y = \lambda 2x$ $\lambda = \frac{2y}{x}$
$4x = \lambda 2y$ $\lambda = \frac{2x}{y}$
$\frac{2y}{x} = \frac{2x}{y}$
$2y^2 = 2x^2$
$y^2 = x^2$
$x = \pm\sqrt{2}$
$y = \pm\sqrt{2}$

h) Determine whether the following series converge or diverge

i) $\sum_{n=1}^{\infty} \frac{n^3 + 1}{5n^3}$

$1 + \frac{2}{4} - \frac{1}{5} +$

[2 marks]

ii) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^5}}$

[2 marks]

QUESTION TWO [20 marks]

a) Write the indefinite decimal number 1.1 2 2 2 2 .... as a rational number [5 marks]

$\frac{11}{90}$ $\frac{101}{90}$

b) Write the general term of each of the following:

i) $\frac{1}{5} + \frac{2}{8} - \frac{3}{11} + \frac{4}{15} - \ldots\ldots\ldots\ldots$

[3 marks]

$3n + 2$

$2n(n+1)$ $2n + 3$ $n^2(n+1)$

$n(n+4)$ $2n(n+1)$ $2n + 3$ $n(n+3)$ $(4n+1)$ $2n(4n-1)$

dw

$\frac{\partial w}{\partial w}$ $dw = \left(\frac{\partial w}{\partial x}\right)dx +$

$\frac{\partial w}{\partial x}$

ii) $\frac{1}{3}, \frac{3}{8}, \frac{5}{15}, \frac{7}{24}, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$ [3 marks]

c) Use he method of differentials to approximate the value of

$\sqrt[3]{(4.9)^2 + (9.8)^2}$ [5 marks]

d) A video tape measuring 220m is to be wound onto a reel of circumference 8.20 cm. Because of the thickness of the tape, each turn is 0.1 cm longer than the previous one. How many turns are required? [4 marks]

QUESTION THREE [20 marks]

a) Find the partial derivatives of the following function: $w = e^{xy^2} + \ln(x+y)$

[4 marks]

b) Calculate the total differential of the function $w = \ln\left(e^{2xz}\right)$ [3 marks]

$\frac{16}{20}$

c) Find the partial derivatives $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ given that,

$x^3 + y^3 + z^3 - x - y - z = 0$ [5 marks]

d) Find $\frac{dz}{dx}$ given that $z = e^{xy}$ where $y = \cos^2 x$ [4 marks]

e) Find $\frac{dy}{dx}$ given that $x^3 + 4x^2y^2 - y^5 = 0$ [4 marks]

QUESTION FOUR [20 marks]

a) Find the equations of the tangent plane and normal line to the surface $z^2 = x^3 - y^2$ at the point $(3,-3,-4)$ [6 marks]

46875

$= x^3 - y^2 - z^2$

$3x^2$ $f_y = -2y$ $f_z = \cancel{-2z}$

$= 27$ $f_y|_m = 6$ $f_z|_m = 8$

tangent plane

$27(x-3) + 6(y+3) + 8(z+4)$

normal

$\frac{x-3}{27} = \frac{y+3}{6} = \frac{z+4}{8}$

$2n-1$

$2n+1$ $n+$

$4n-1$ $5,$ $5n-2$

1)

y.

$f(x,y,z) = \sin\frac{1}{(x,y,z)}$ (0,0)

$f(x,y,z) = (xyz)^{-1}$

$\frac{\partial f}{\partial x} = \frac{1}{2x}\; x^2+3x$

b) Locate and classify all critical points of the function

$$f(x,y)=2y^4+2y^2-x^3-3x^2y$$ [8 marks]

$f_x = -3x^2-6xy \to x+2y=0$

$f_y = 8y^3+4y-3x^2 \to 8y^3+4y-3x^2=0$

c) Use Lagrange multipliers to find the maximum and minimum values of the function $f(x,y)=4x^2y$ subject to the constraint $x^2+y^2=3$

[6 marks]

$8xy$ $g_x = 2x$ $\lambda = 4y$ $4y^2 = 2x^2$ $x^2+\frac{x^2}{2}=3$

x^2 $g_y = 2xy$ $\lambda = \frac{2x^2}{y}$ $2y^2 = x^2$ $\frac{3}{2}x^2 = 3$

$4y = \frac{2x^2}{y}$ $2y^2+y^2=3$ $x^2 = \frac{1}{2}$

$y^2 = 1$ $x = \pm\sqrt{0.5}$

$y = \pm 1$

QEUSTION FIVE [20 marks]

a) Evaluate $\iint_R f(x,y)\,dA$ where $f(x,y)=e^{x+y}$ and R is the region bounded by y = x, x = 0 and y = 1 [6 marks]

b) Find the volume for the region beneath the surface $z = 4+x^2+y^2$ and above the domain bounded by y = 0, x = 0 and x+y = 1 [7 marks]

$\int_0^1 \frac{3x^2}{5+x^3}\,dx = \ln(5+x^3)\Big|_0^1$

c) Evaluate the following iterated integral

$$\int_0^1\int_0^{x^2}\frac{3}{5+x^3}\,dy\,dx$$ [3 marks]

$\iint_0^{x^2}\frac{3}{5+x^3}\,dy\,dx = \int_0^1\left(\frac{3y}{5+x^3}\right)\Big|_0^{x^2}dy = \int\frac{3x^2}{5+x^3} = \int\frac{3x^2}{u}\frac{du}{3x^2}$

d) Find a function f(x,y) that satisfy the following conditions

$$f_x = -\frac{1}{yx^2}-\frac{2}{y^2x^3}$$

$$f_y = -\frac{1}{xy^2}-\frac{2}{y^3x^2}$$

$$f(1,1) = -3$$ [4 marks]

$(x,y,z)^{-1-1}$

$\frac{\partial f}{\partial x} = \frac{yz}{-(x,y,z)^2}$

$8y^3+4y=0$

$2y^3+y=0$

$y(2y^2+1)=0$

$2y^2+1=0$

$2y^2=-1$

$y^2=-\frac{1}{2}$

$\frac{yz}{(xyz)^2}$

EGERTON UNIVERSITY

UNIVERSITY EXAMINATIONS

NJORO CAMPUS

THIRD SEMESTER 2009/20010

THIRD YEAR EXAMINATION FOR THE DEGREES OF BACHELOR OF EDUCATION (SCIENCE), BACHELOR OF SCIENCE (WEEN, ICEN, MENT, AGEN AND GEN)

MATH 318 – CALCULUS III

STREAM: BSC. BED (SC) **TIME:** 2 HRS

DAY/TIME: FRIDAY, 12.00 – 2.00 P.M. **DATE:** 11/12/2009

INTRUCTIONS

Attempt question **ONE** and any **other TWO** questions

QUESTION ONE (30 MARKS)

a) Evaluate the following limits

i) $\lim\limits_{n\to\infty}\left(\sqrt{n+2}-\sqrt{n}\right)$ (3 marks)

ii) $\lim\limits_{n\to\infty}\dfrac{2^n}{n!}$ (4 marks)

b) Find the sum of the series $\sum\limits_{n=1}^{\infty}\dfrac{4}{10^n}$ (3 marks)

c) Find the partial derivatives of the following functions

i) $f(x,y)=y\sin xy$ (2 marks)

ii) $w=\ln\left(x+y+z^2\right)$ (3 marks)

d) Calculate the total differential of $f(x,y,z)=\frac{xy}{z}$ (3 marks)

e) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ of the equation $x^2+2y^2+3z^2+yz+y=0$. (4 marks)

f) Determine the critical points of $f(x,y)=x^3-3x+y^2+4y$ (4 marks)

g) Evaluate the following integral $\int_0^4 \int_{\frac{1}{2}y}^{\sqrt{y}} 6xy\,dxdy$ (5 marks)

QUESTION TWO (20 MARKS)

a) Express the number 1.4444... as a rational number (4 marks)

* b) Using the ratio test determine the interval of convergence of the following power series $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(x-2)^n}{2^n n}$ (4 marks)

c) Determine whether or not the following series converges and state the method used

$\frac{1}{(n+1)^2+3(n+1)} \cdot \frac{n^2+3n}{1}$

* i) $\sum_{n=1}^{\infty}\left(\frac{n}{n+1}\right)^{n^2}$ (4 marks)

ii) $\sum_{n=1}^{\infty}\frac{1}{n^2+3n}$ (4 marks)

* d) Show that the following improper integral converges and compute its value $\int_0^3 \frac{dx}{\sqrt{9-x^2}}$ (4 marks)

$64 - 3 \times 16$

48

64
48
16

QUESTION THREE (20 MARKS)

a) Evaluate the following iterated integrals

i). $\int_0^2 \int_0^x \frac{1}{x^2+1} dy\, dx$ (4 marks)

ii) $\int_0^1 \int_0^\pi \int_0^\pi y \sin z\, dx\, dy\, dz$ (4 marks)

b) Find the volume of solid between the surface $z = \cos(2x+5y)$ and above the domain bounded by x = y, x = 3y, y = 0 and y = π (7 marks)

c) Evaluate $\iint_R f(x,y)\,dA$ where R is the region bounded by y = x^2, y = 4, x = 0 and x = 1 and $f(x,y) = 2xy$ (5 marks)

QUESTION FOUR (20 MARKS)

* a) Find the equations of the tangent plane and normal line to the surface $z - x^3 - 3x^2y^4 = 7$ at the point where x = -1 and y = -1. (10 marks)

b) Determine the absolute maximum and minimum values of $f(x,y) = 2 + 2x + 2y - x^2 - y^2$ on a triangular plate in the first quadrant bounded by the lines x = 0, y = 0 and y = 9 – x. (10 marks)

QUESTION FIVE (20 MARKS)

a) Using the method of Lagrange multipliers, find the maximum and minimum values of the functions $f(x,y) = x+y$ on the circle $x^2+y^2=1$ (10 marks)

b) Find the approximate change in the hypotenuse of a right angled triangle of base 15 cm and height 20 cm when base is lengthened by $\frac{5}{8} cm$ and height is shortened by $\frac{5}{16} cm$. (5 marks)

c) Find the function $f(x,y)$ satisfying the conditions

$f_x = 6x + 12, \quad f_y = 9y^2 - 8y, \quad f(-1, 2) = 8$ (5 marks)

END*

c) Calculate the partial derivatives of the function

$f(x,y,z)=4y\sin z-3x^2z$ [4 marks]

d) Evaluate the following iterated integral $\int_{-2}^{2}\int_{0}^{y}(x^3-1)\,dx\,dy$ [3 marks]

e) Determine the critical points of the function $f(x,y)=x^3-3xy+y^3$

[4 marks]

f) Evaluate the total differential of the function $z=(x^2-4y^2)^3$ [4 marks]

g) Find the rational number represented by the infinite decimal number 1.323232..................... [4 marks]

h) Determine the value of $\lim_{n\to\infty}\frac{(n-2)(2n-3)}{n^2}$ [3 marks]

QUESTION TWO: [20 marks]

a) Calculate the partial derivatives of the following

i) $f(x,y,z)=\frac{2}{\sqrt{x^2+y^2+z^2}}$ [3 marks]

ii) $f(x,y)=4^{\frac{x}{y}}-\frac{y}{x}$ [4 marks]

b) Calculate the total differentials of the functions

i) $f(x,y)=\sqrt{x^2+y^2}$ [2 marks]

ii) $f(x,y)=2^{xy}$ [2 marks]

a) Use the method of differentials to approximate the value of

$\left((2.01)^2 + (2.97)^2\right)^3$ 2.127, 2.778 [5 marks]

b) Find $\frac{dy}{dx}$ given that $x^2 + 4xy^2 - y^5 = 0$ [4 marks]

QUESTION THREE [20 MARKS]

a) Find the equations of the tangent plane and normal line to the surface $z^2 = x^2 - y^2$ at the point $(5, -3, -4)$ [6 marks]

b) Locate and classify all critical points of the function $f(x, y) = x^3 - 2y^2 - 2y^4 + 3x^2y$ Remind. [8 marks]

c) Find the maximum and minimum values of $f(x, y) = xy$ subject to $x^2 + y^2 = 1$ [6 marks]

QUESTION FOUR (20 MARKS)

a) Evaluate the following interacted integrals

i) $\int_0^1 \int_0^{y^2} \frac{3}{4 + y^3}\, dx\,dy$ [4 marks]

ii) $\int_0^2 \int_0^{4y} (x + 2y)\, dx\,dy$ [4 marks]

b) Find the volume of the region beneath the surface $z = 4 - x^2 - y^2$ above the domain bounded by y = 0, x = 0 and x + y = 1 [7 marks]

$$\frac{3}{5}-\frac{5}{8}+\frac{7}{11}-\frac{9}{14}+\ldots\ldots$$ (3 marks)

(f) Determine the critical points of $f(x,y)=x^3-3x+y^2+4y$. (4 marks)

(g) Determine whether the following series converge or diverge.

(i) $\sum_{n=1}^{\infty}\frac{n^2+1}{5n^2}$

(ii) $\sum_{n=1}^{\infty}\frac{1}{n^2\sqrt{n}}$ (4 marks)

(h) Determine the value of

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$$ (3 marks)

QUESTION TWO (20 MARKS)

(a) Evaluate the following iterated integrals.

(i) $\int_0^2\int_0^x \frac{1}{x^2+1}\,dydx.$ (4 marks)

(ii) $\int_0^{\sqrt{\pi}}\int_0^y Sin\left(y^2\right)dxdy$ (4 marks)

(b) Find the volume of solid between the surface $z=\cos(2x+5y)$ and the domain bounded by x = y, x = 3y, y = 0 and y = π. (7 marks)

(c) Evaluate $\iint_R f(x,y)dA$ where R is the region bounded by $y = x^2$, y = 4, x = 0 and x = 1 and f(x,y) = 2xy. (5 marks)

QUESTION THREE (20 MARKS)

(a) Write down the equation of the tangent plane and the normal to the surface $f(x, y) = x^3 + 3x^2y^4 + 7$ at the point where x = -1 and y = -1. (6 marks)

(b) Determine and classify all the critical points of the function $f(x, y) = y^3 + 3yx^2 - 3y^2 - 3x^2$

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EGERTON UNIVERSITY

UNIVERSITY EXAMINATIONS
SECOND SEMESTER 2004/2005

THIRD YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE

MATH 318 – CALCULUS III

STREAM: B.SC , BED (SC) **TIME:** 2 HRS

DAY: FRIDAY, 12 – 2.00 P.M. **DATE:** 29/7/2005

INSTRUCTIONS:

1. Answer question ONE and any other two questions.
2. Write well organized solutions.

QUESTION ONE (30 MARKS)

(a) Find the partial derivatives of the following functions.

(i) $f(x,y)=\sqrt{\frac{y^2}{x+1}}$ (2 marks)

(ii) $W=\ln(x+y+z)$ (3 marks)

(b) Evaluate the following

$$\int_0^2\int_y^3 (x+y)\,dxdy$$ (4 marks)

(c) Calculate the total differential of
$f(x,y,z)=Sin\,xyz$ (3 marks)

(d) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ of the equation
$x^2+2y^2+3z^2+yz+y=0$ (4 marks)

(e) Write down the general term of the series below, (Hint: Critical points are four). (8 marks)

(c) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ given that

$x^3 + 2y^3 + z^3 - 3xyz - 2y + 3 = 0.$ (6 marks)

QUESTION FOUR (20 MARKS)

(a) (i) State Taylor's series expansion of f(x, y) about the point (a, b). (4 marks)

(ii) Expand the function sin (x + y) in powers of x and $y - \frac{\pi}{2}$. (6 marks)

(b) By method of differentials, approximate the value of $\left(\sqrt{15} + \sqrt{99}\right)^2$. (5 marks)

(c) Find the function f(x, y) satisfying the following conditions.

$F_x = 6x + 12,\ f_y = 9y^2 - 8y,\ f(-1, 2) = 8.$ (5 marks)

QUESTION FIVE (20 MARKS)

(a) Express the decimal number 1.22222...... as a rational number. (4 marks)

(b) Use the ratio test to determine convergence or divergence of the series,

$\frac{1}{9} + \frac{2!}{9^2} + \frac{3!}{9^3} + \frac{4!}{9^4} + \ldots\ldots + \frac{n!}{9^n} + \ldots\ldots$ (4 marks)

(c) Find the formula for the n^{th} partial sum S_n and then compute the sum of the infinite series

$$\sum_{n=1}^{\infty} \frac{1}{(n+1)(n+2)}$$ (6 marks)

(d) Show that the following series n convergent and find its sum

$$\sum_{n=1}^{\infty} \left(-\frac{1}{3}\right)^n$$ (6 marks)

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EGERTON UNIVERSIT

UNIVERSITY EXAMINATIONS

SECOND SEMESTER 2002/2003

THIRD YEAR EXAMINATION FOR THE DEGREES OF BACHEL
SCIENCE AND BACHELOR OF EDUCATION

MATH 318 – CALCULUS III

STREAMS: AGEN, WEEN, B.SC. (GEN), B.ED. TIME: 2 H

DAY: TUESDAY 8.30 – 10.30 AM DATE: 5/0

INSTRUCTIONS:

Answer question one and any other two questions.

QUESTION ONE (30 MARKS)

(a) Find the partial derivatives of the following functions:

(i) $f(x,y) = \sqrt{\frac{y}{x+1}}$

(ii) $z = x\sin x^2y$ (4 ma

(b) Evaluate the following:

$$\int \int [illegible] (x+z)\,dx\,dy$$ (4 ma

(c) Calculate the total differential of:

$z = x\tan xy$ (3 mar

(d) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ of the equation

$x^2 - 2y^2 + 3z^2 - yz + y = 0$ (4 mark

(e) Write down the general term in the following

$$\frac{3}{5} + \frac{5}{8} + \frac{7}{11} + \frac{9}{14} + \dots$$ (4 marks)

(f) Determine the critical points of $f(x,y) = x^3 - 3x - y^2 + 4y$ (4 marks)

(g) Determine the value of

$$\lim_{n \to \infty} \frac{(1-n)(2-n)(3-n)}{n^3}$$ (3 marks)

(h) Determine whether the following series converge or diverge:

(i) $\sum_{n=1}^{\infty} \frac{n^2-1}{4n^3}$

(ii) $\sum_{n=1}^{\infty} \frac{n^2+1}{n\sqrt{n}}$ (4 marks)

QUESTION TWO (20 MARKS)

(a) Write down the equation of the tangent plane and the normal to the surface $f(x,y) = x^3 + 3x^2y^4 + 7$ at the point where $x = 1$ and $y = -1$. (6 marks)

(b) Determine and classify all the critical points of the function:

$$f(x,y) = y^3 - 3yx^2 - 3y^2 - 3x^2$$ (7 marks)

(c) Find the point on the circle $x^2+y^2 = 1$ for which $x + 2y$ is a maximum. (7 marks)

QUESTION THREE (20 MARKS)

(a) Evaluate the following iterated integrals:

(i) $\int_0^1 \int_0^x \frac{1}{x^2+1}\, dy\, dx$

(ii) $\int_0^{\sqrt{\pi}} \int_0^x \sin(x^2)\, dy\, dx$ (6 marks)

(b) Find the volume of the region beneath the surface $z = 2x + 3y^2$ and above the domain bounded by $y = \sqrt{x}$, $y = 0$ and $y = x - 2$ (7 marks)

(c) Evaluate $\iint_R f(x,y)\, dA$ where R is the region bounded by $x = y$, $x = 3y$, $y = 0$ and $y = \pi$ and $f(x,y) = \cos(2x + 5y)$ (7 marks)

QUESTION FOUR (20 MARKS)

(a) (i) Express sinx in Maclaurin's series. (5 marks)

(ii) Show that:

$$\frac{x}{\sin x} = 1 + \frac{1}{6}x^2 + \frac{7}{360}x^4 + \dots$$ (5 marks)

(iii) Estimate the value of:

$\int_0^1 \frac{\sin x}{x}\, dx$ to three decimal places. (5 marks)

(b) Use Taylor's approximation formula to estimate the value of

$\sin(29.89)^\circ$

(Use 3 terms only and leave you in seven decimal places). (5 marks)

QUESTION FIVE (20 MARKS)

(a) Find the function $f(x,y)$ satisfying the following conditions:

$f_x = 6x + 12$, $f_y = 12y^2 - 12y$

and $f(-1, 2) = 4$. (5 marks)

(b) Use the method of differentials to approximate the value of

$\frac{\sqrt{25}}{\sqrt[5]{30}}$ to 4 decimal place. (5 marks)

(c) Sketch the level curves for the function

$f(x,y) = x^2 - y$, with $c_n = n$, $n = 1, 3, 5$.

(level curves are given by $f(x,y) = C_n$). (5 marks)

(d) Suppose (a,b) is a saddle point of a function $f(x,y)$. If $f_{xy}(a,b) = 6$ and $f_{xx}(a,b) = 2$,

What are the possible values of $f_{yy}(a,b)$? (5 marks)

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EGERTON UNIVERSITY

UNIVERSITY EXAMINATIONS

SECOND SEMESTER 2003/2004

THIRD YEAR EXAMINATION FOR THE DEGREES OF BACHELOR OF EDUCATION, BACHELOR OF SCIENCE AND BACHELOR OF ARTS

MATH 318 –CALCULUS III

STREAMS: BED, BSC, B.A. TIME: 2 HRS

DAY: WEDNESDAY, 8.30 – 10.30 A.M. DATE: 4/5/2005

INSTRUCTIONS:

Answer question ONE and any other two questions.

QUESTION ONE (30 MARKS)

(a) Describe the loci of the points of discontinuity of the functions.

(i) $z = \frac{1}{2x - y + 4}$ (3 marks)

(ii) $z = \ln(2x + y)^{x}$. (3 marks)

(b) Given that $u = x + \frac{y-x}{z-y}$, calculate the value of $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$ (5 marks)

(c) Find df at the point (-3, -4, 5) if $f(x,y,z) = \frac{z}{(x^2+y^2)^{\frac{1}{2}}}$ (5 marks)

(d) Calculate the following limits

(i) $\lim_{n \to \infty} \sqrt[n]{2^{n+1}}$ (2 marks)

(ii) $\lim_{n \to \infty} \left(\frac{1}{n}\right)^{\frac{1}{n}}$ (3 marks)

(e) Write the equation of the tangent plane and the equations of the normal to the surface $z = x^2 + y^2$ at the point where $x = 1$ and $y = 3$. (5 marks)

(f) Evaluate the values of the following:

$$\int_{-1}^{1} xy\sqrt{x^2y+4}\,dx$$ (4 marks)

QUESTION TWO (20 MARKS)

(a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ if

$$(x^2+y^2)^{4} - 3(x^2+y^2) = 5$$ (5 marks)

(b) Find $\frac{dz}{dx}$ and $\frac{dz}{dy}$ if

$$x^2 - 2y^2 + 3z^2 - yz + y = 0.$$ (5 marks)

(c) Find $\frac{dz}{dx}$ and $\frac{dz}{dy}$ if

$x = u\cos V$, $y = u\sin V$, $z = V$. (5 marks)

(d) Find $\frac{du}{dt}$ if $u = xyz$ and $x = t^2 + 1$, $y = \ln t$, $z = \tan t$. (5 marks)

QUESTION THREE (20 MARKS)

(a) Find the stationary points of the function $f(x,y) = x^3 + 3xy^2 - 15x - 12y$ and identify their nature. Calculate the value of f_{max} and f_{min} if any. (10 marks)

(b) Find the minimum and maximum values if any of the function $f(x, y) = xy$ under the constraint $4x^2 + 9y^2 = 36$. (10 marks)

QUESTION FOUR (20 MARKS)

(a) Investigate the convergence or divergence of the following series:

(i) $\sum_{n=1}^{\infty} \frac{e^{-n}}{n+1}$ (2 marks)

(ii) $\sum_{n=1}^{\infty} \frac{n+n^2}{n^2-4}$ (3 marks)

(b) Find the limits of the following sequences if they do converge.

(i) $\left(\frac{2^n+1}{e^n-1}\right)$ $\left(\frac{2^n+1}{e^n-1}\right)$ (2 marks)

(ii) $\left(\frac{2-n^2}{3+n^2}\right)^n$ $\left(\frac{2-n^2}{3+n^2}\right)^n$ (2 marks)

(c) Calculate the partial derivatives of the function

$W = e^{[illegible]}$ (6 marks)

(d) Find $f(x, y)$ satisfying the following.

$$f_x = 2x - \frac{3y^2}{x^2}, \quad f_y = \frac{6y}{x} + 7, \quad f(1,0) = 0$$ (5 marks)

QUESTION FIVE (20 MARKS)

(a) Write the general term in each of the following series.

(i) $\frac{3}{4} + \frac{5}{4^2} + \frac{7}{4^3} + \ldots\ldots$

(ii) $\frac{1}{2} - \frac{1}{6} + \frac{1}{12} - \frac{1}{20} + \ldots\ldots$ (4 marks)

(b) Use geometric series to write the indefinite decimal number 0.121212... as a rational number. (4 marks)

(c) Use the method of differentials to approximate the value of $(1.03)^3(0.99)^2$. (4 marks)

(d) Find the volume beneath the surface $f(x, y) = 4 + y$ and above the domain bounded by the equation $(y-3)^2 = x - 1$ and $x = 5$. (8 marks)

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MATH318 CALCULUS111

PROBLEM SET 1

Q1 Calculate the following limits

$$(a)\ \lim_{(x,y)\to(0,0)} \frac{3x^2-y^2+5}{x^2+y^2+2} \qquad (b)\ \lim_{(x,y)\to(0,\ln 2)} e^{x-y} \quad e$$

$$(c)\ \lim_{(x,y)\to(0,0)} \cos\left(\frac{x^2+y^2}{x+y+1}\right) \qquad (d)\ \lim_{(x,y)\to(1,1)} \ln\left|1+x^2y^2\right|$$

Q2 By considering different paths determine whether the following functions have limits or not

$$(a)\ f(x,y)=\frac{x}{\sqrt{x^2+y^2}} \qquad (b)\ f(x,y)=\frac{x^4}{x^4+y^2}$$

$$(c)\ f(x,y)=\frac{x+y}{x-y} \qquad (d)\ f(x,y)=\frac{x^4-y^2}{x^4+y^2}$$

Q3 Find all the partial derivatives of the following functions

$$(a)\ f(x,y)=\frac{x}{x^2+y^2} \qquad (b)\ f(x,y)=\frac{x+y}{xy-1}$$

$$(c)\ f(x,y,z)=(x^2+y^2+z^2)^{\frac{1}{2}}$$

$$(d)\ f(x,y,z)=x-\sqrt{y^2+z^2}$$

Q4 Show that the following functions satisfies the Laplace Equation

$$f(x,y,z)=\left(x^2+y^2+z^2\right)^{-\frac{1}{2}}$$

Q5. Find all points (x,y) where $f(x,y)$ has a possible maximum/minimum use 2^{nd} test to determine nature of $f(x,y)$ at these points,

$$(a)\ f(x,y)=-2x^2+2xy-y^2+4x-6y+5$$

$$(b)\ f(x,y)=x^2+2xy+5y^2+2x+10y-3$$

$$(c)\ f(x,y)=x^3-2xy+4y$$

Q6. Find the possible values of (x,y,z) at which $f(x,y,z)=2x^2-3y^2+z^2-2x-y-z$ assumes its extreme values.

Q7 Find the possible values (x, y, z) which the function $f(x, y, z) = 5 + 8x - 4y + x^2 + y^2 + z^2$ assumes its extreme values.

Q8 Find and identify the nature of the critical points of the function

$$f(x, y) = \ln x + \ln y - xy$$

Q9. Find the dimensions of a rectangular box with maximum volume that has total surface area of $600cm^2$

Q10. A rectangular box without a lid is to have fixed volume 4000cm what dimension would minimize its total surface area?

Q11. The sum of 3 positive numbers is 120. What is the maximum possible value of their product?

(Hint: Constraint $x + y + z = 120$ and Maximize the product)

Q12. Find the derivative of $f(x, y) = x \tan^{-1}(y/x)$ at the point $P(0,1)$ in the direction $\underline{a} = 2\underline{i} - \underline{j}$

Q13. Find $\frac{dw}{dt}$ given that $w = \ln(x^2 + y^2 + z^2)$, $x = \cos t$, $y = \sin t$ and $z = 4\sqrt{t}$

Q14. Find $\frac{\partial w}{\partial p}$ and $\frac{\partial w}{\partial q}$ if $w = xy + yz + zu + uv$, $x = pq$, $y = q\tan^{-1} p$, $z = \ln(q - 3p)$, $u = \sqrt{q + 5p}$, $v = p + q + 1$

Q15. Find $\frac{\partial z}{\partial y}$ and $\frac{\partial z}{\partial x}$ given that $z^3 - xy + yz + y^3 - 2 = 0$