EGERTON UNIVERSITY

UNIVERSITY EXAMINATIONS

MAIN CAMPUS
ACADEMIC YEAR 2012/2013

THIRD YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE (SCIENCE)

MATH 325 - NUMERICAL NALYSIS I

STREAM: BSC **TIME:** 2 HRS

DAY: **DATE:**

INSTRUCTIONS

- Answer Question ONE and any other TWO of the remaining Questions
- Clearly show all your workings in the answer booklet provided
- Use of Scientific calculator is allowed.

QUESTION ONE (30 MARKS) COMPULSORY

a) Given that n_0 is an approximate value of the square root of n and that $x = \frac{n}{{n_0}^2} - 1$. Show that the square root of n is given by $\sqrt{n} = n_0\left(1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} - \frac{5x^4}{128} + \cdots\right)$. Hence find $\sqrt{7}$ correct to 5 significance figures if $n_0 = 2.5$ (5mks)

b) Given that $y = x^3 + 2x^2 - 3x + 2$, calculate the values of y for; $x = 0, 1, 2, 3, 4$ and form a forward difference table. Find the value of $y = 5$ by extending the table and verify that the same value is obtained by substitution (5mks)

c) Use Taylor series method to solve the differential equation $\frac{dy}{dx} = -xy,\ y(0) = 1$ (5mks)

d) Apply Lagrange formula to find $f(x)$ given that $f(0) = -6,\ f(2) = 10,\ f(4) = 50,$ (5mks)

e) By applying Runge- Kutta formula of order 2, approximate the value of y when $x = 1.1$, given that $\frac{dy}{dx} = 3x + y^2$ and $y = 1.2$ when $x = 1$ (5mks)

f) Use Simpsons rule to approximate the value of $\int_0^1 \frac{dx}{x+1}$ with 2n=4 giving your answer to 4 decimal places (5mks)

QUESTION TWO (20 MARKS)

a) Find a forward difference representation for $\frac{df}{dx}$ which is of $o(h^2)$ (9mks)

b) Locate the error in the following entries and correct it; 1.203, 1.424, 1.681, 1.992, 2.379, 2.848, 3.429 and 4.136 (4mks)

c) Given the table below, calculate $f(1.72)$ correct to 6 decimal places using Newton's forward difference formula

x	1.7	1.8	1.9	2.0
$f(x)$	0.3979849	0.3399864	0.2818186	0.2238908

(7mks)

QUESTION THREE (20 MARKS)

a) Use Runge- Kutta fourth order method to find $y(0.2)$ with $h = 0.1$ for the initial value problem $\frac{dy}{dx} = \sqrt{x+y},\ y(0) = 1$ (15mks)

b) Use Trapezoidal rule to compute $\int_0^3 \frac{dx}{16+x^2}$ with $n = 6$, giving your answer to 3 decimal places (5mks)

QUESTION FOUR (20 MARKS)

a) Apply Modified Euler method to solve $\frac{dy}{dx} = x + 3y$ subject to $y(0) = 1$ and hence find an approximate value of y when $x = 1$ (10mks)

b) Using Newton Raphson method, evaluate to 4 decimal places the root of $e^x = 3x$ which lies between 0 and 1 (10mks)

QUESTION FIVE (20MARKS)

Find the coefficients a, b, c, d, m, n and p in order that the Runge-Kutta formula below form a Taylor series through the term in h^4

$$k_1 = hf(x, y)$$

$$k_2 = hf(x + mh, y + mk_1)$$

$$k_3 = hf(x + nh, y + nk_2)$$

$$k_4 = hf(x + ph, y + pk_3)$$

$$y(x + h) - y(x) \approx ak_1 + bk_2 + ck_3 + dk_4$$

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