EGERTON UNIVERSITY

UNIVERSITY EXAMINATIONS

NJORO CAMPUS

SECOND SEMESTER 2012/2013

THIRD YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN ENGINEERING

MATH 327: METHODS 1, MATH

STREAM: B.Sc, Eng, Y3 **TIME:** 2 HOURS

DAY: MONDAY, 8.30 – 11.30 A.M **DATE:** 13/05/2013

INSTRUCTIONS

QUESTION ONE-30 marks-COMPUSORY

(a) Given the Taylor series for $\cosh x$ and $\sinh x$ as $\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + ... + \frac{x^{2n}}{(2n)!}$; $\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + ... + \frac{x^{2n+1}}{(2n+1)!}$, (n=0…∞) obtain a corresponding series for $\tanh x$ **(5 marks)**

(b) Maximize the function Show that if $x^2 e^y$ subject to the constraint then $x^2 + y^2 = 3$ **(6 marks)**

(c) (i)What is the main feature of a periodic function with period T? **(1 mark)**

For a T-periodic function, prove that $\int_{\alpha}^{\beta} f(x)dx = \int_{\alpha+T}^{\beta+T} f(x)dx$ **(3 marks)**

(ii) Determine the period, T, for the function $f(x) = 2\cos(2x)$ **(3 marks)**

(d) Using a power series approach, obtain a solution for the first order O.D.E. $y' + 2xy = \frac{1}{1-x}$ about an ordinary point $x = 0$. [Hint: use the series expansion for $\frac{1}{1-x} = \sum_{0}^{\infty} x^n$, find c up to C_6] **(8 marks)**

(e) Find the Laplace transform of $F(t) = \sin(at)$ **(5 marks)**

QUESTION TWO-20 marks-OPTIONAL

(a) (i) Define *ordinary point*, *regular* and *irregular* singular point (i.r.s.p) for an order 2 differential equation $a_0(x)y'' + a_1(x)y' + a_2(x)y = 0$ at a point x_0. **(5 marks)**

(ii)Classify the nature of the singularities of the equation $(x-1)(x+1)^3 x^2 y'' + x\,y' - 2y = 0$ **(5 marks)**

(b) (i) Define the radius of convergence of a power series $y = \sum_{n=0}^{\infty} c_n (x-x_0)^2$ about x_0 **(2 marks)**

(ii) Find the radius of convergence of the Taylor series for $y = \frac{1}{1+x^2}$ (Hint: you may begin reasoning from the well known expansion for $y = \frac{1}{1+x}$, hint in Q1(d),or binomial theorem) **(3marks)**

(c) Use series to solve the order 1 differential equation $y' + 2y = 0$ about the ordinary point $x_0 = 0$ **(5 marks)**

QUESTION THREE-20 marks-OPTIONAL

(a) Prove the orthogonal property that $\int_{-\pi}^{\pi} \sin(nx)\cos(mx)dx = 0, \forall\, m,n$ **(4marks)**

(b) (i) Fourier series involves expressing a periodic function as a trigonometric sum to infinity, as in, $f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty}[a_n \cos(n\omega_0 x) + b_n \sin(n\omega_0 x)]$. If T is the period of this function, prove that

$$\frac{1}{2}a_0 = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(x)dx,\; a_n = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(x)\cos(m\varpi_0 x)dx \text{ and } b_n = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(x)\sin(m\varpi_0 x)dx$$ **(7 marks)**

(ii)Sketch, and find the Fourier coefficients for the function $f(x) = \left\{\begin{matrix} 0, -\pi < x < 0 \\ \frac{x}{\pi},\; 0 < x < \pi \end{matrix},\; f(x) = f(x+2\pi)\right\}$ and hence the corresponding Fourier Series. **(9 marks)**

QUESTION FOUR-20 marks-OPTIONAL

(a) Use the method of separation of variables to obtain the solution $U(x,t)$ of the one-dimensional wave equation $U_{xx} - \frac{1}{c^2}U_{tt} = 0$, given the initial conditions $U(0,t) = 0$ and $U(l,t) = 0$ for all t. **(10 marks)**

(b) Use the method of Frobenius to solve the equation $x^2 y'' + 5x\,y' + (x+4)y = 0$ about a point of singularity $x_0 = 0$ **(10 marks)**

QUESTION FIVE-20 marks-OPTIONAL

(a) . A hot water storage tank is a vertical tank closed on the top by a hemisphere of the same diameter. Its volume is 400 cubic meters. Determine the total height and diameter the tank has to have in order to minimize surface heat loss. Evaluate that volume. (i.e. the minimum surface area possible with that volume) **(7 marks)**

(b) Determine the Fourier series for the function $f(x)=x^2$ in the interval $[-\pi,\pi]$. Substituting $x=\pi$, show that $\sum_{n=1}^{\infty}\frac{1}{n^2}=\frac{\pi^2}{6}$ [Hint: you'll employ integration by parts, take period as 2π] **(8 marks)**

(c) Determine the <u>inverse</u> Laplace transform of $\frac{\frac{2}{7}}{5s^2-\frac{2}{5}s+9}$ **(5 marks)**

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