**KIMATHI UNIVERSITY COLLEGE OF TECHNOLOGY**
**University Examination 2010/2011**

SECOND YEAR ***SUPPLEMENTARY/SPECIAL*** EXAMINATION FOR BACHELOR OF SCIENCE IN **ACTUARIAL SCIENCE**

**SMA 2370 : CALCULUS IV**

**DATE: 8TH November 2010** **TIME: 8.30 AM – 10.30 AM**

========================================================================

**INSTRUCTIONS:**

(a) **Answer Q1 (Compulsory) and ANY OTHER TWO (2) questions.**
(b) **DO NOT answer more than [3] questions in total. Use of sketches where appropriate**

## QUESTION ONE-30 marks-COMPUSORY

(a) Figure out and sketch the graphs of the entities below in $\Re^3$: (i) $z = \sqrt{4 - x^2 - y^2}$ **(4 marks)**

(ii) $2x + 3y + 4z - 12 = 0$ **(4 marks)**

(b) Find the directional derivative of the scalar function $\Phi(x, y, z) = x^2 yz = xyz + 4xz^2$ at the point $(1,2,3)$ in the direction $2\vec{i} + \vec{j} + \vec{k}$ **(3 marks)**

(c) (i) Prove the property of orthogonal functions that $\int_{-\frac{T}{2}}^{\frac{T}{2}} \sin^2(m\omega_0 t)dt = \frac{T}{2}$ **(3 marks)**

(ii) Find the period of the function $f(t) = (10\cos t)^2$ **(3 marks)**

(d) Establish whether the function $f(t) = \int_1^{\infty} \sin(\frac{1}{t})dt$ converges or diverges **(4 marks)**

(e) Find the equation of the tangent line and normal plane to the space curve given parametrically as $x = t - \cos t,\ y = 3 + \sin 2t$, $z = 1 + \cos 3t$ **(6 marks)**

Along the straight line joining the points $(0,0,0)$ and $(1,1,1)$ find the line integral $\int_C \vec{f}.d\vec{r}$ given that

$\vec{f} = (3x^2 + 6y)\vec{i} - 14yz\vec{j} + 20xz^2\vec{k}$ **(3 marks)**

## QUESTION TWO-20 marks-OPTIONAL

(a) Sketch the square function of the square function below

$$f(t) = -2 \quad -1 < t < 0$$
$$f(x) = 2 \quad 0 < t < 1$$
$$f(t) = f(t+2)$$
**(3 marks)**

And determine its Fourier Series **(6 marks)**

(b) Given the scalar operator $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$, find $\nabla^2(\frac{1}{r})$ where $r = \sqrt{x^2 + y^2 + z^2}$

i.e. magnitude of $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ . **(6 marks)**

(c) Find the turning points of the function $u(x,y,z) = x^2 + 2y^2 + z$ subject to the constraint $\Phi(x,y,z) = x^2 - z^2 - 2 = 0$ **(5 marks)**

## QUESTION THREE-20 marks- OPTIONAL.

(a) Evaluate $\iint \vec{B}.\vec{N}\, dS$ where S is the outward surface of the cube bounded by $x = 0, x = 1, y = 0, y = 1$ and $z = 0, z = 1$ without using Gauss' Divergence Theorem **(10 marks)**

(b) Now use Gauss' Theorem to establish the result of (a) above **(5 marks)**

(c) Optimize $x^2 e^y$ subject to $x^2 + y^2 = 3$ using Lagrange's multipliers **(5 marks)**

## QUESTION FOUR-20 marks-OPTIONAL

(a) Convert the equations $(i)$ $4x^2 + 4y^2 - z^2 = 0$ and $(ii)$ $z = 2\sqrt{x^2 + y^2}$ into polar coordinates **(4 marks)**

(b) A solid is formed by the hemisphere $z = \sqrt{4 - x^2 - y^2}$ on the sides and the cone $z = x^2 + y^2$ drilled from the center. Use spherical coordinates (or otherwise) to find its volume **(6 marks)**

(c) Use Green's Theorem to evaluate $\oint_C (y - \sin x)dx + \cos x dy$ given that C is the path of the triangle PQR, $P(0,0), Q(\frac{\pi}{2},0).R(\frac{\pi}{2},1)$ . **(5 marks)**

(d) Evaluate $\iiint_V \nabla.\vec{f}\, dv$ given that $\vec{f} = (2y - 3z)\vec{i} + 2xy\vec{j} - 4x\vec{k}$ and the implied volume is bounded by $x = 0, x = 2$ , $y = 0, y = 6$ and $z = x^2, z = 4$ **(5 marks)**

## QUESTION FIVE-20 marks- OPTIONAL

(a) Given $z = x^2 - y^2$ , $x = r\cos\theta$ , $y = r\sin\theta$, determine $z_r, z_\theta, z_{\theta\theta}$ and $z_{r\theta}$ **(5marks)**

(ii) Find and classify all the turning points of $f(x,y) = 2x^2 - y^3 - 2xy$ **(5 marks)**

(b) Given vector $\vec{X} = z(x + y^2)\vec{i} + z(z + x^2)\vec{j} - x^2 y(x + y)\vec{k}$ prove that $\vec{X}.curl\,\vec{X} = 0$ **(5 marks)**

(c) Prove the MVT, i.e. $\int_a^b f(x)dx = (b-a)f(c)$ and find the mean value of $\int_2^5 x^2 dx, x \in [2,5]$ **(5 marks)**

**Best wishes, sincerely!**