KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2009/2010

INSTITUTIONAL BASED PROGRAMME

EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE

SMA 305: COMPLEX ANALYSIS I

DATE: THURSDAY, 2ND SEPTEMBER 2010 **TIME: 2.00 P.M. - 4.00 P.M.**

INSTRUCTIONS: Answer question ONE and any other TWO questions.

QUESTION ONE - Compulsory (30 marks)

(a) If $z = 1 + \sqrt{3i}$, find z^5. (3 marks)

(b) Determine whether or not the function $f(z) = \begin{cases} \dfrac{z^2+9}{z-3i} , & if \quad z \neq 3i \\ 2+3i , & if \quad z = 3i \end{cases}$

is continous at $z = 3i$ (3 marks)

(c) Differentiate $f(z) = 2z^3 + z$ using first principles. (4 marks)

(d) Show that $f(z) = e^{-y}e^{ix}$ is analytic. (3 marks)

(e) Find the linear transformation that maps $z_1 = 1$, $z_2 = 0$, $z_3 = -1$

onto $w_1 = i$, $w_2 = \infty$, $w_3 = 1$ respectively. (3 marks)

(f) Evaluate $\int_c \bar{z} dz$ from $z = 0$ to $z = 4 + 2i$

and then from $z = 2i$ to $z = 4 + 2i$ (4 marks)

(g) Find the residues of $f(z) = \dfrac{z}{(z-2)(z+2)^2}$

at all its poles. (3 marks)

(h) Expand $f(z) = (z-3)\sin\dfrac{1}{z+5}$ into a Laurent Series at $z = -5$. (3 marks)

(i) Evaluate $\oint_c \dfrac{dz}{z^2 - iz + 6}$ around the circle $c : |z - 2i| = 1$ (4 marks)

QUESTION TWO (20 marks)

(a) Prove that $\cos 4\theta = 8\sin^4\theta - 8\sin^2\theta + 1$ (5 marks)

(b) Find the third roots of $z = -1 + i$ (5 marks)

(c) Express $\sin^5\theta$ in multiple angles. (5 marks)

(d) Show that the function

$$f(z) = \begin{cases} \dfrac{z^2 + 2iz}{z^2 - iz + 6}, & if \ z \neq -2i \\ 4i, & z = -2i \end{cases}$$

is not continous at $z = -2i$ (5 marks)

QUESTION THREE (20 marks)

(a) Prove that $f(z) = z^3 - 2z$ is harmonic. (5 marks)

(b) Given that the function $u(x, y) = y^3 - 3x^2 y$ is harmonic, find v(x,y) such that $f(z) = u + iv$ is analytic. (5 marks)

(c) Show that $\sin z = \sin x \cosh y + i \cos x \sinh y$. (6 marks)

(d) Find the principle value of $(-1)^i$ (3 marks)

QUESTION FOUR (20 marks)

(a) Find the image of the unit square in the z plane given by a transformation

$f(z) = Bz + c$ where

$B = 3 + 4i$

$c = 2 + 3i$

(5 marks)

(b) Find the fixed points of $f(z) = \dfrac{3z - 10}{z + 1}$ (5 marks)

(c) Prove that $f(z) = \dfrac{1}{z}$ maps the line $X = K$ onto a circle. (5 marks)

(d) Evaluate the Integral $\int_{(0,3)}^{(2,4)} (2y + x^2)(3x - y)dy$ along the parabola

$X = 2t,\ y = t^2 + 3.$

QUESTION FIVE (20 marks)

(a) State without proof

(i) Cauchy's Integral Theorem (2 marks)

(ii) Cauchy's Residue Theorem (3 marks)

(b) Expand $f(z) = \dfrac{1}{(z+1)\ (z+3)}$ in a Laurent Series valid for

$0 < |z + 1| < 2$ (5 marks)

(c) Evaluate $\oint_c \dfrac{e^z}{(z-2)\,(z-4)}\ dz$ when

(i) $|z| = 5$ (6 marks)

(ii) $|z| = 3$ (4 marks)

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