KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2010/2011

FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF EDUCATION

SMA 305: COMPLEX ANALYSIS I

DATE: Monday, 29th November, 2010 TIME: 4.30 p.m. – 6.30 p.m.

INSTRUCTIONS:

Answer question **ONE** and any other **TWO** questions.

QUESTION 1 (30 MARKS)

a) Solve the equation $z^3 - 4\sqrt{3} + 4i = 0.$ (4 marks)

b) Show that the function $f(z) = z^2 + 3iz + 8 - 7i$ satisfies the Cauchy Riemann equations. (5 marks)

c) Determine the singular points of $f(z) = \dfrac{4z+3}{z^2+6z+13}$ and the derivative $f'(z)$ at all other points (5 marks)

d) Evaluate $\dfrac{6i^{22} + i^{31}}{(3+i)(2-3i)}$ (4 marks)

e) Find the residues at the poles of $f(z) = \dfrac{z^2+9}{z^3+2z^2+2z}$ hence evaluate

$\displaystyle\int_C \frac{z^2+9}{z^3+2z^2+2z}\, dz$ where C is a circle enclosing all the poles (6 marks)

f) Evaluate $\displaystyle\int_{1+2i}^{3-2i} (4x^2 + y)\, dx + (5y - 2x)\, dy$ along the straight line joining $1+i$ *to* $3-2i.$ (5 marks)

QUESTION 2 (20 MARKS)

a) Use the polar form of complex numbers to show that

$$\left(\frac{\sqrt{3}-i}{\sqrt{3}+i}\right)^4\left(\frac{1+i}{1-i}\right)^5 = -\frac{\sqrt{3}}{2}-\frac{1}{2}i.$$ (6 marks)

b) Show that $\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$

$$\sin 5\theta = \left[16\cos^4\theta - 12\cos^2\theta + 1\right]\sin\theta.$$ (7 marks)

c) Prove that if z_1 and z_2 are complex numbers then

$|z_1 + z_2|^2 = |z_1|^2 + 2\operatorname{Re} z_1\overline{z_2} + |z_2|^2$ and use it to prove that $|z_1 + z_2| \leq |z_1| + |z_2|$

(7 marks)

QUESTION 3 (20 MARKS)

a) Prove that $\cos^{-1} z = \frac{1}{i} In\left(z + \sqrt{z^2 - 1}\right)$. Find the general and principal values of

$\cos z = i$ (6 marks)

b) Evaluate $\lim_{z \to 0} \frac{1 - \cos z}{z^2}$ (3 marks)

c) Show that $\cos^2 z + \sin^2 z = 1$ (4 marks)

d) Given $u = (x\cos y - y\sin y)e^x$

i) Show that u is harmonic (2 marks)

ii) Find the harmonic conjugate v of u (3 marks)

iii) Write $f(z) = u + iv$ in terms of z (2 marks)

QUESTION 4 (20 MARKS)

a) Evaluate $\int_C \left(2x+iy^2\right) dz$ along the curve $y=x^3$ from $(1,1)$ to $(2,8)$.

(8 marks)

b) If $f(z)$ is analytic inside and on the boundary C of a simply connected region R, prove that if z_o is any point inside R then $f(z_o)=\frac{1}{2\pi i}\int_C \frac{f(z)}{z-z_o}\, dz$

(7 marks)

c) Use Cauchy's integral formula in (b) above to evaluate $\int_C \frac{3z^2+5}{(z-2)(z+5)} dz$ where C is the circle $|z|=4$. (5 marks)

QUESTION 5 (20 MARKS)

a) State Cauchy's integral formula for the *n*th derivative. Use it to evaluate $\int_C \frac{e^{3z}+z^3}{(z-2)^4}\, dz$ where C is the circle $|z|=3$ (6 marks)

b) Determine the Lawrent series for $f(z)=\frac{13-z}{z^2-z-6}$ valid on $2<|z|<3$.

(7 marks)

c) Find the residues of the function $f(z)=\frac{2z^2+1}{(z-1)^2(z^2+9)}$ at its poles hence evaluate $\int_C \frac{2z^2+1}{(z-1)^2(z^2+9)}\, dz$ where C is the circle $|z|=5$. (7 marks)