



KENYATTA UNIVERSITY
UNIVERSITY EXAMINATIONS 2010/2011
INSTITUTE OF OPEN LEARNING

**EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE AND
BACHELOR OF EDUCATION**

SMA 305: COMPLEX ANALYSIS 1

DATE: TUESDAY 1ST FEBRUARY 2011 TIME: 11.00 A.M – 1.00 P.M

INSTRUCTIONS:

Answer **question ONE** and any other **TWO**.

QUESTION ONE (30 MARKS)

a) Solve the equation $Z^4 + 16i = 0$.

Plot the values of Z in an argand diagram. [7 marks]

b) Given that $f(Z) = Z^3 - 2Z^2 - (4 + 5i)$ find $f(Z_0)$ if $Z_0 = 3 - i$ [5 marks]

c) Evaluate the limit

$$\lim_{Z \rightarrow 2i} \frac{Z^2 + 4}{2Z^2 + (3 - 4i)Z - 6i} \quad [4 \text{ marks}]$$

d) Let $f(Z) = u + iv$ be an analytic function. If $u = x^3 - 3x^2y$ find $f(Z)$. [5 marks]

e) Expand $f(Z) = \frac{Z}{(1+Z)(3+Z)}$ into a Laurent series for $1 < |Z| < 3$. [5marks]

f) Define a harmonic function and proof that $x^2 - y^2 + 2xy$ is a harmonic function. [4 marks]

QUESTION TWO (20 MARKS)

a) Determine the region of convergence of the following series.

i)
$$\sum_{n=1}^{\infty} \frac{(Z+2)^{n-1}}{(n+1)^3 4^n}$$
 [5 marks]

ii)
$$\sum_{n=0}^{\infty} \frac{(Z-3i)^n}{2^n (n+1)^2}$$
 [5 marks]

b) i) State without Proof the Cauchy's integral formula. [4 marks]

ii) Use the formula stated in (i) above to evaluate $\oint_c \frac{Z}{(9-Z^2)(Z+i)} dZ$ $c: |Z|=2$
[6 marks]

QUESTION THREE (20 MARKS)

a) State without proof the Cauchy's integral formula. [3 marks]

b) Use the formula above to evaluate

i)
$$\oint_c \frac{e^{2Z}}{(Z+1)^4} df \quad c: |Z|=3$$
 [6 marks]

ii)
$$\oint_c \frac{Z}{(9-Z^2)(Z+i)} df \quad c: |Z|=2$$
 [6 marks]

c) Prove that the function $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$ satisfies the Laplace equation [5 marks]

QUESTION FOUR (20 MARKS)

a) Find the Laurent Series of the function $f(Z) = \frac{Z}{(Z+1)(Z+2)}$ at the point where $Z=-1$. [5 marks]

b) Verify that the real and imaginary parts of $f(Z) = w = u + iv = Z^2 + 5iZ + 3 - i$ Satisfy the Cauchy- Riemann equations. [6 marks]

c) Find the Cube root of Unity and hence factorise the expression. [6 marks]

d) If $u(x, y) = 3x^2 + 2x - y^3 - 2y^2$, find $V(x, y)$ the harmonic conjugate of u. [3 marks]

QUESTION FIVE (20 MARKS)

- a) Determine the linear fractional transformation T that maps $Z_1 = 1$
 $Z_2 = 0, Z_3 = -1$ into $w_1 = i, w_2 = \infty$ and $w_3 = 1$. [6 marks]
- b) Let $Z_1 = r_1 e^{i\theta_1} = Z_2 = r_2 e^{i\theta_2}$ find
- (i) $Z_1 Z_2$ [3 marks]
- (ii) Z_1 / Z_2 [3 marks]
- c) Show that $f(Z) = \frac{3Z^4 - 2Z^3 + 8Z^2 - 2Z + 5}{Z - i}$
is not continuous at $Z = i$ [4 marks]
- d) Show that $\lim_{Z \rightarrow \infty} \frac{2Z + 1}{Z^4 + 1} = 2$ [4 marks]