KIMATHI UNIVERSITY COLLEGE OF TECHNOLOGY

University Examinations 2011/2012

FOURTH YEAR FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN **ACTUARIAL SCIENCE**

SMA 2431 TIME SERIES ANALYSIS

DATE: 16TH AUGUST 2011 TIME: 2 HOURS

Instructions: Answer QUESTION ONE and any other TWO QUESTIONS.

QUESTION ONE (30 marks) (COMPULSORY)

(a). State and describe briefly the periodic changes in a time series. **[4 marks]**

(b). Determine whether the process $12X_t = 10X_{t-1} - 2X_{t-2} + 12\epsilon_t - 11\epsilon_{t-1} + 2\epsilon_{t-2}$ is both stationary and invertible. State the significance of these properties in time series analysis. **[6 marks]**

(c). Given that $X_t = 0.5X_{t-1} + \epsilon_t$ and ϵ_t is a white noise with mean zero and variance σ_ϵ^2. Show that find the process is the sum of infinite moving average process. Find its autocorrelation function. **[8 marks]**

(d). The values 10.4, 13.2 and 35.6 corresponding to time $t_1 = 2, t_2 = 5$ and $t_3 = 8$ respectively were selected from a time series. Fit a modified exponential curve to this time series and predict the series value at $t_4 = 15$. **[6 marks]**

(e). Distinguish between the *white noise process* and the *random walk process*.
. **[4 marks]**

(f). Give an expression for $X_t - 5X_{t-1} + 7X_{t-2} - 3X_{t-3}$ in terms of second order differences. **[2 marks]**

QUESTION TWO (20 marks) (Optional)

(a). An actuary has calculated the variable rate of return on money market funds for the last nine months as follows:

Month:	1	2	3	4	5	6	7	8	9
Rate:	6.2	5.8	6.5	6.4	5.9	5.9	6.0	6.8	6.5

(i). Calculate a 4-point centered moving average. **[4 marks]**

(ii). Calculate the MSE for the moving average method. **[2 marks]**

(iii). Calculate the forecast rate of return for the next two months using exponential smoothing with $\alpha = 0.2$. **[4 marks]**

(b). Find the effect of a linear filter on the original time series, $X_t = e^{i\lambda t}$. **[10 marks]**

QUESTION THREE (20 marks) (Optional)

(a). The data below gives the average quarterly prices of a commodity in four years:

Year	Q1	Q2	Q3	Q4
2007	40.3	44.8	46.0	48.0
2008	30.1	53.1	53.3	59.5
2009	47.2	54.1	52.1	55.2
2010	55.4	59.0	61.6	65.3

Using the multiplicative model,

(i) Compute the quarterly seasonal indices.

(ii) Compute the normalized quarterly seasonal indices **[10 marks]**

(b). Explain the importance of invertibility in a moving average process. State the sufficient condition for a MA(q) process to be invertible and determine whether $X_t = \epsilon_t + 0.4\epsilon_{t-1} - 0.2\epsilon_{t-2}$ is invertible. Hence or otherwise find its autocorrelation function. **[10 marks]**

QUESTION FOUR (20 marks) (Optional)

(a). Explain is the significance of the spectral density function in time series analysis. Let $X_t = \epsilon_t - 5\epsilon_{t-1} + 6\epsilon_{t-2}$. Find the spectral density function of X_t. **[6 marks]**

(b). For the stationary AR(2) process:

$$X_t = \frac{5}{6}X_{t-1} - \frac{1}{6}X_{t-2} + \epsilon_t$$

find the general form for the autocorrelation function. **[14 marks]**

QUESTION FIVE (20 marks) (Optional)

(a). Find the autocorrelation function of the stationary zero mean ARMA(2,1) process:

$$X_t = 1.3X_{t-1} - 0.4X_{t-2} - 0.4\epsilon_{t-1} + \epsilon_t$$

[13 marks]

(b). For an AR(1) process given by $X_t = \alpha X_{t-1} + \epsilon_t$, derive an expression for k-steps ahead forecast and find the variance of the forecast error. **[7 marks]**