KIMATHI UNIVERSITY COLLEGE OF TECHNOLOGY

University Examination 2011/2012

FOURTH YEAR FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE

SMA 2436: STOCHASTIC PROCESSES

DATE: 17TH AUGUST 2011 TIME: 8.30AM – 10.30AM

Instruction: Answer Question ONE and any other TWO

QUESTION ONE (30 MARKS)

a). Briefly discuss the following terms as used in stochastic processes.

i. Markovian process

ii. Random experiment

iii. Homogeneous Poisson process **[6 marks]**

b). Consider the "Memoryless property", suppose that X has an exponential distribution, such that, $P(X > s+t \mid X > t) = P(X > s)$ for all $s,t \geq 0$. Prove this. **[5 marks]**

c). Find the probability generating function of a Geometric distribution, hence find;

i. Mean **[4 marks]**

ii. Variance. **[5 marks]**

d). Suppose that a Markov Chain $\{X_t, t \geq 0\}$ whose state space is the set {0,1,2}, has the one-step transition probability matrix P given by,

$$P = \begin{bmatrix} 13/36 & 11/54 & 47/108 \\ 4/9 & 4/27 & 11/27 \\ 5/12 & 2/9 & 13/36 \end{bmatrix}$$

i. Is the matrix P stochastic? If 'yes' why? **[1 mark]**

ii. Find $P(X_3 = 1, X_1 = 2, X_0 = 1)$ if $P(X_0 = i) = 1/3$. **[4 marks]**

e). Discuss the concept of Geometric Brownian process and counting process. **[3 marks]**

f). What do you understand by the term '*Branching process*es? **[2 marks]**

<u>QUESTION TWO (20 MARKS)</u>

a). Explain the term Non-homogeneous Poisson process? **[5 marks]**

b). Suppose that the failures of a certain machine occur according to a Poisson process with rate $\lambda = 2$ per week and that exactly two failures occurred in the interval [0,1]. Let $t_0 (> 3)$ be an arbitrary value of t. What is the probability that, at time t_0 (at least) two weeks have elapsed since;

i). The last failure occurred? **[2 marks]**

ii). The penultimate failure occurred? **[2 marks]**

iii). What is the probability that there will be no failures during the two days beginning with to if exactly one failure occurred (in all) over the last two weeks? **[3 marks]**

c). A particle move one step to the right with probability p and one step to the left with probability $1-p$ in one unit time with absorbing barriers at 0 and at n. If the particle is at state r, then by using a transition probability matrix, represent this simple random walk model that will describe the position of a particle after n steps. **[4 marks]**

d). In a departmental store one cashier is there to serve the customers. And the customers pick up their needs by themselves. The arrival rate is 9 customers for every 5 minutes and the cashier can serve 10 customers in 5 minutes. Assuming Poisson arrival rate and exponential distribution for service rate, find:

i. Average number of customers in the system. **[1 mark]**

ii. Average number of customers in the queue or average queue length. **[1 mark]**

iii. Average time a customer spends in the system. **[1 mark]**

iv. Average time a customer waits before being served. **[1 mark]**

<u>QUESTION THREE (20 MARKS)</u>

a). Distinguish between 'Stationary' and 'Independent' increments as used in stochastic processes; **[4 marks]**

b). The arrivals and services follow a Poisson process. Customers arrive at a rate of 8 per hour and the service rate is 10 per hour.

i. What is the average number of customers waiting for service? **[5 marks]**

ii. What is the average time for a customer to be in the system? **[2.5 marks]**

iii. What is the average time a customer must wait in the queue? **[2.5 marks]**

c). We define the stochastic process $\{X(t), t \geq 0\}$ by;

$X(t)=B(t) | \{B(t) \geq 0\}$ for $t \geq 0$, where $\{B(t), t \geq 0\}$ is a standard Brownian motion.

Find E(X) hence verify if the stochastic process $\{X(t), t \geq 0\}$ is Gaussian. **[6 marks]**

QUESTION FOUR (20 MARKS)

a). State the elements of the queuing system. **[3 marks]**

b). Suppose that a Markov Chain $\{X_t, t \geq 0\}$ whose state space is the set {1,2,3}, has the one-step transition probability matrix P given by,

$$P = \begin{bmatrix} 65/180 & 55/270 & 235/540 \\ 8/18 & 8/54 & 22/54 \\ 10/24 & 4/18 & 26/72 \end{bmatrix}$$

Suppose that the initial distribution is given by $\alpha_i = P(X_0 = i) = \{1/2, 1/3, 1/6\}$. Find;

i. $P(X_2 = 2)$ **[2 marks]**

ii. $P(X_0 = 1, X_2 = 3)$ **[3 marks]**

iii. $P(X_1 = 2, X_2 = 3, X_3 = 1 \mid X_0 = 1)$ **[5 marks]**

iv. $P(X_{12} = 1 \mid X_5 = 3, X_{10} = 1)$ **[2 marks]**

v. $P(X_3 = 3, X_5 = 1 \mid X_0 = 1)$ **[5 marks]**

QUESTION FIVE (20 MARKS)

a). State the Birth and Death Process. **[4 marks]**

b). Let Z={1,2,3,4,5} be the state space of a Markov chain with a transition matrix;

$$P = \begin{bmatrix} 0.2 & 0 & 0.5 & 0.3 & 0 \\ 0.1 & 0 & 0.9 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0.4 & 0.1 & 0.2 & 0 & 0.3 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Represent the above matrix on a transition graph. **[5 marks]**

c). Suppose that X_1, X_2 and X_3 are independent random variables, all of which have Exp (λ) distribution. Calculate the $P[X_1 < (X_2 + X_3)/2]$. **[8 marks]**

d). Define a martingale. **[3 marks]**