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University Examinations 2012/2013

SECOND YEAR, SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOROF SCIENCE IN INFORMATION TECHNOLOGY

SMA 2101: CALCULUS 1

DATE: APRIL 2013 **TIME: 2 HOURS**

INSTRUCTIONS: *Answer question **one** and any other **two** questions*

QUESTION ONE (30 MARKS)

a) Find all values of k that make the function f(x) continuous on the interval $(-\infty, \infty)$. (4 Marks)

$$f(x) = \begin{cases} k^2x^2 - 3x - 1, & if\ x < 1 \\ 3k\cos(x-1), & if\ x \geq 1 \end{cases}$$

b) Evaluate;

$\lim_{x\to 27} \frac{x^{\frac{1}{3}}-3}{x-27}$ (3 Marks)

c) Given that $y = x^2\cos x + x$, find $\frac{dy}{dx}$. (3 Marks)

d) Find the equation at the tangent line to the curve $y = x^2(x+4) at\ x = 1$. (4 Marks)

e) Given that $x = 2\sin\theta$ and $y = \cos 2\theta$, show that $\frac{d^2y}{dx^2} = -1$. (4 Marks)

f) If $2y^3 - y + 3x - 2 = 0$, find $\frac{dy}{dx}$ in terms of x and y. (3 Marks)

g) Find the coordinates of the turning points on the curve $y = 3x^3 + 6x^2 + 3x - 1$ and distinguish between them. (5 Marks)

h) Evaluate $\int_1^2 (x^2-1)(x+2)dx$ (4 Marks)

QUESTION TWO (20 MARKS)

a) Find the derivatives of the following functions and simplify your answer

i. $y = \frac{\sin x + \cos x}{\sin x - \cos x}$ (4 Marks)

ii. $y = \ln\left(\frac{x^2-4}{x^2+6}\right)$ (5 Marks)

b) Given that $y = \frac{5}{x^2}$, determine the change in y if x changes from 2.50 to 2.52. (3 Marks)

c) The equation of a curve is given as $y^4 = 4x^4 + 6xy$

i. Find the equation of the tangent line to this curve at (1,2). (4 Marks)

ii. Find the equation of the normal line at (1,2). (2 Marks)

d) A missile fired from ground level rises y metres vertically upwards in t seconds according to the law $y = 100t - \frac{25}{2}t^2$. Find the time when the height is maximum. (2 Marks)

QUESTION THREE (20 MARKS)

a) Compute $\lim_{x\to\infty} \frac{3x^2-7x+2}{1-x^3}$. (3 Marks)

b) A function is represented parametrically by the equations

$x = 2t + 3t^2$

$y = t^2 + 2t^3$

Find $\frac{dy}{dx}$ and hence show that these equations satisfy the equation $y = (\frac{dy}{dx})^2 + 2(\frac{dy}{dx})^3$. (5 Marks)

c) Given that $lny + \frac{x}{y} = 4$, show that $\frac{dy}{dx} = \frac{-y}{y-x}$. (4 Marks)

d) Determine the constant B such that f(x) is continuous at x=4 where,

$$f(x) = \begin{cases} 3x + B, & if\ x \leq 4 \\ \frac{x^2-16}{x-4}, & if\ x > 4 \end{cases}$$ (4 Marks)

e) Find $\frac{dy}{dx}$ if $y = \log_3(x^2 + 4)$. (4 Marks)

QUESTION FOUR (20 MARKS)

a) Find points on the curve $y = 2x^3$, where the tangent line is parallel to the line $y = 13x + 5$. (4 Marks)

b) The height of an object moving vertically upwards given by the equation

$y = -16t^2 + 96t + 111$, where y is in metres and t is in seconds.

i. Find the velocity when t=0 (2 Marks)

ii. Calculate its maximum height. (3 Marks)

iii. Calculate velocity when y=0. (3 Marks)

c) The price of a certain commodity in dollars per unit at time t (measured in weeks) is given by $P = 8 + 4e^{-2t} + te^{-2t}$.

i. Find the price of the commodity at t=0. (1 Mark)

ii. Find the equilibrium of the commodity. (3 Marks)

iii. Calculate the rate of change of the price of the commodity at t=0. (4 Marks)