University Examinations 2012/2013

FIRST YEAR, FIRST SEMESTER EXAMINATIONS FOR THE DEGREE OF
BACHELOR OF SCIENCE IN COMPUTER SCIENCE AND BACHELOR OF BUSINESS
INFORMATION TECHNOLOGY AND SECOND YEAR, FIRST SEMESTER FOR THE
BACHELOR OF SCIENCE IN COMPUTER TECHNOLOGY

SMA 2100: DISCRETE MATHEMATICS

DATE: AUGUST 2012   **TIME: 2 HOURS**

INSTRUCTIONS: *Answer question **one** and any other **two** questions*

QUESTION ONE (30 MARKS)

a) In a survey of 100 students, the numbers studying the various languages were found
   to be: Spanish 28, German 30, French 42, Spanish and German 8, Spanish and French
   10, German and French 5, all three languages 5.
   i.   How many were studying no language?
   ii.   How many had French as their only language?   (5 Marks)

b) Give the converse, contra positive and inverse of the following implication: "if it rains
   today, I will go to college tomorrow".   (6 Marks)

c) Write the following statements in Symbolic form:
   i.   The sun is bright and the humidity is not high.
   ii.   If you do not see me tomorrow, it means I have gone to Chicago   (4 Marks)

d) Consider functions $f: A \to B$ and $g: B \to C$. Prove the following:
   i.   If f and g are one – one, then the composition $g0f$ is one – one.
   ii.   If f and g are onto functions, then gof is an function.   (5 Marks)

e) Prove that if $a \mid b \ and \ a \mid c$, then $a \mid (bx + cy)$ where $x, y \epsilon$ Z.   (5 Marks)

f) Show that $1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6} \ for \ n \geq 1$ by mathematical induction.
                                                                     (5 Marks)

QUESTION TWO (20 MARKS)

a) Test the validity of the following argument. If two sides of a triangle are equal, then
   the opposite angles are equal. Two sides of a triangle are not equal. The opposite
   angles are not equal.   (6 Marks)

b) Given the following propositions: p: food is good, q: service is good, r : restaurant is 5 Star. Write the following in Symbolic notation:
   i. Either food is good or the service is good or both
   ii. Either food is good or service is good but not both
   iii. Food is good while the service is poor.
   iv. It is not the case that food is good and the rating is five star
   v. If the food and the service are good, then the rating will be 5 star.
   vi. It is not true that 5 star rating always means good food and good service.
   (6 Marks)

c) Construct the truth table for: $(a \lor b) \leftrightarrow [((\sim a) \land C) \rightarrow (b \land c)]$. (8 Marks)

d) Write the following statement in symbolic form: there will be no test examination tomorrow if Professor is out of town or there's a Matatu Strike. (3 Marks)

QUESTION THREE (20 MARKS)

a) Find the inverse of the following functions:
   i. $f(x) = \frac{x+1}{x}$
   ii. $f(x) = 4e^{(6x+2)}$ (7 Marks)

b) Show that these identities hold:
   i. $x \oplus y = (x + y)(\overline{xy})$
   ii. $x \oplus y = (\overline{xy}) + (\bar{x}y)$ (5 Marks)

c) Translate the distributive law $x(y + z) = xy + xz$ into logical equivalence and prove. (8 Marks)

QUESTION FOUR (20 MARKS)

a) For each pair of integers a and b, find integers q and r such that $a = bq + r$ for $0 < r \leq |b|$.
   i. $a = -381$ and $b = 14$
   ii. $a = -433$ and $b = -17$ (8 Marks)

b) Solve each linear congruence equation:
   i. $3 \equiv 2mod\ 8$
   ii. $6x \equiv 5mod\ 9$ (6 Marks)

c) Find the smallest positive integer x such that when x is divided by 3 it yields a remainder 2, when x is divided by 7 it yields a remainder 4, and when x is divided by 10 it yields a remainder 6? (6 Marks)

QUESTION FIVE (20 MARKS)

a) Prove that for any integer $n \geq 1, 2^{2n} - 1$ $is\ divisible\ by$ 3. (6 Marks)

b) A certain store sells envelopes in packages of five and packages of 12 and you want to buy n envelopes. Prove that for every $n \geq 44$, this store can sell you exactly n envelopes assuming an unlimited supply of each type of envelope package.(8 Marks)

c) Let $g : R \to R$ be defined by $g(x) = x^2$.
  i.   What is the domain of g
  ii.   What is the range of g
  iii. Is g onto?
  iv.   Is g one to one.

Give reasons for your answers above.   (6 Marks)