

QUESTION ONE

Suppose the demand function for corn is  $Q_d = 10 - 2p$ , and supply function is  $Q_s = 3p - 5$ . The government is concerned that the market equilibrium price of corn is too low and would like to implement a price support policy to protect the farmers. By implementing the price support policy, the government sets a support price and purchases the extra supply at the support price. In this case, the government sets the support price  $p_s = 4$ .

- a) Calculate the original market equilibrium price and quantity in absence of the price support policy. (3marks)

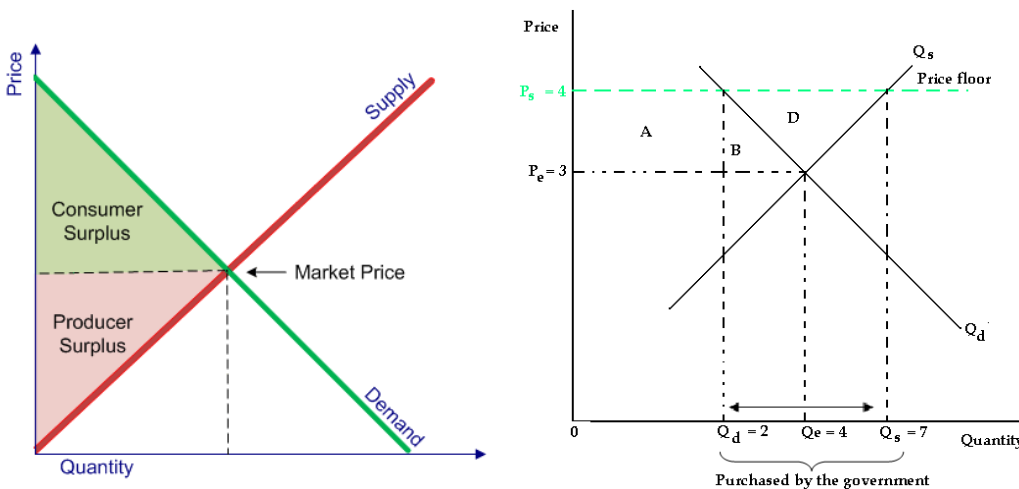
**Answer: The original equilibrium price is  $P_e = 3$ . And the equilibrium quantity is 4.**

- b) At the support price  $p_s = 4$ , find the quantity supplied by the farmers, the quantity demanded by the market, and the quantity purchased by the government. (4 marks)

**Answer: When the price is  $p_s = 4$ , the demand is 2, and supply will be 7, so the government need to buy**

- c) Draw a diagram to show the change in the producer surplus due to the implementation of the price support policy. Calculate the change in the producer surplus (4 marks).

**Answer: The producer's gain is  $1 \times 7 - \frac{1}{2}(7 - 4) \times 1 = 11/2$  which is area A+B+D in the following graph.**



- d) Draw a diagram to show the change in the consumer surplus due to the implementation of the price support policy. Calculate the change in the consumer surplus (4 marks)

**The loss to consumer is  $2(4-3) + (4-2)(4-3)/2 = 3$ , which is area A+B in the following graph.**

- e) Calculate the cost to the government to implement the price support policy. Draw a diagram to show the government cost. (5 marks)

**Answer: Cost to the government:  $P_s Q_g = 20$ , which is the rectangle area indicated in the above graph by Gov's purchase\*price.**

- f). Suppose now the government switches from price support policy to subsidy policy. For each unit of corn produced, the government subsidizes the farmer  $s = 5/3$ . Find the new equilibrium price under this subsidy policy. How much money will the government have to spend in order to implement this subsidy policy? (10 marks)

**Answer: Under subsidy  $s = 5/3$ , the suppliers face the price  $P_b + 5/3$ , and the consumers face the price  $P_b$ , thus,  $10 - 2P_b = 3(P_b + s) - 5$ . This implies  $P_b = 2$ . The new amount of supply is  $10 - 2P_b = 6$  which is the amount that the government needs to subsidize. The government's total budget is  $sQ (2*5) = 10$ .**

## QUESTION TWO

Suppose that a large percentage of the micro-motors used to adjust the rear-view mirror in every car are made by Mabuchi; a mid-size Japanese Manufacturer which has a monopoly in the market for these micro-motors. In order to produce  $Q$  micro-motors, it costs Mabuchi  $C(Q) = 2Q^2$ .

- a) Find the marginal cost of producing a micro-motor for Mabuchi [2 marks].

$$MC = 4Q \text{ [2marks]}$$

- b) If the demand for micro-motors is  $P = 48 - 4Q_d$  Find;

- c) The total revenue [1 mark]

$$TR = PQ = Q [48 - 4Q_d] = 48Q - 4Q^2 \text{ [2marks]}$$

- d) The marginal revenue [1mark]

$$MR = 48 - 8Q \text{ [1 mark]}$$

- e) The level of output that maximizes Mabuchi's profits [3marks]

$$\begin{aligned} MR &= MC \\ 48 - 8Q &= 4Q \\ Q &= 4 \text{ [@ 1 mark]} \end{aligned}$$

f) State the price Mabuchi is charging at the profit maximizing output level [1 mark]

$$P = 48 - 4Q = 32 \text{ [1 mark]}$$

g) Find the level of output would maximize total surplus in the micro-motor's market [4 marks] [Hint: The Total surplus is maximized when price equals Marginal Cost]

$$P = MC \text{ [ 2 marks]}$$

$$48 - 4Q = 4Q; \text{ [1 mark]} \quad Q = 6 \text{ [1 mark]}$$

h) Suppose the government knew the demand and production functions. Find a price regulation the government could impose that would induce Mabuchi to maximize total surplus, i.e., produce the efficient quantity from part [iii] [4 marks]

**Substitute  $Q=6$  in  $P=48-4Q$  to find the price that corresponds to a demand of 6 units which is 24 [2marks]. The Japanese government should set a regulated price of  $P = 24$ , because Mabuchi would face a constant marginal revenue function and would choose to produce where  $MC = 24$ , i.e., at the efficient level where  $P=MC$  [2 marks]**

i) If the government subsidized Mabuchi for every unit of micro-motors produced, what quantity would Mabuchi choose as a function of  $s$ ; the subsidy? Find the choice of subsidy that maximizes total surplus, i.e., induces Mabuchi to produce the efficient quantity from part [iii] [6 marks]

**Marginal revenue will be  $MR = 48 - 8Q + s$ . [M2] If Intel equates marginal revenue to marginal cost, then**

$$4Q = 48 - 8Q + s \text{ [M1]}$$

$$Q = 4 + s/12 \text{ [M1]}$$

**Solving for  $s$  with  $Q = 6$ , [M1] we find that a subsidy of 24 [A1] per unit is required.**

j) Both the price regulation policy from part [iv] and the subsidy policy from part [v] maximize total surplus. Is there any reason one policy might be preferred over the other [ 2marks]

**The subsidy increases Mabuchi's profits [1mark] at the expense of government [taxpayers]; the price regulation redistributes some of Mabuchi's profits to consumers [1 mark].**

**[Total 20 marks]**

### QUESTION THREE

Suppose the market demand and market supply for a commodity are given as follows:

$$Q_d = 100 - 2p$$

$$Q_s = 4 + 4p$$

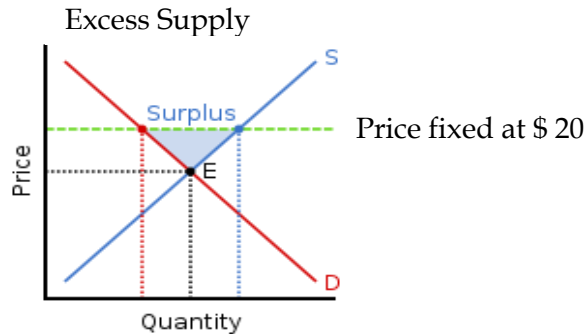
i. Determine the equilibrium price and quantity for the market. [2 marks]

Equating  $Q_d$  to  $Q_s$ ,  $P_e=16$  and  $Q_e= 60$ [@1 mark]

- ii. Suppose the maximum price is fixed at \$20. Illustrate the effect of this price control on a supply and demand diagram. Is there a shortage? If so, what is the excess demand? [3marks]

In this case the  $Q_d$  would be 60[1mark] and  $Q_s$  would be 84[1mark]. There would be no shortage but instead an excess supply of 16[1mark]

Or diagrammatically



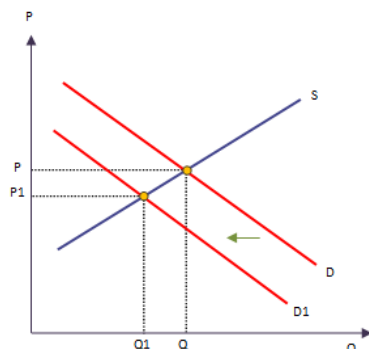
Must show  $Q_d$  [1mark] and  $Q_s$  [1mark] on the x-axis[ well labeled 1mark] incase of using a diagrammatic approach

- iii. Calculate the change in consumer and producer surplus, dead weight loss resulting from this price control

**Solution: Consumer Surplus: compute the change in area below the demand curve but above the price. Producer Surplus: compute the change in area above the supply curve but below the price**

- iv. The popularity of brown ‘chapatis’ causes consumers’ tastes to shift away from bread. Show the effect of this shift on the market demand for blue band, which is used mainly when people eat toast. [2 marks]

The demand market demand for blue band will shift inwards [decrease]

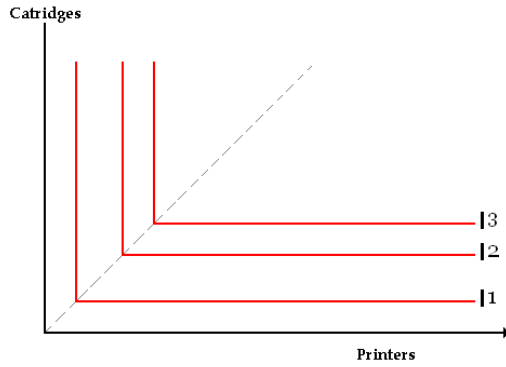


Demand for Blue band will shift inwards from  $D$  to  $D_1$  as a result of change in consumer preferences to brown ‘chapatis’

Quantity of Blue band demanded[@ 1 mark]

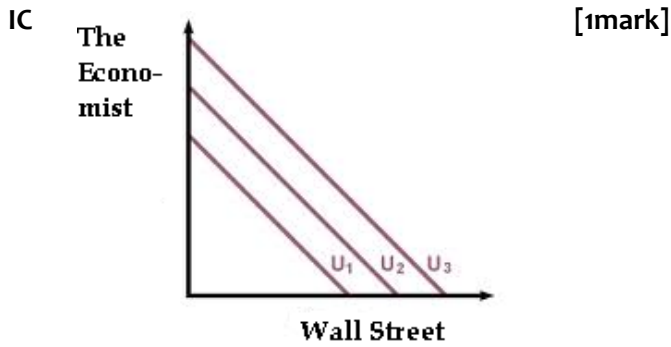
- v. Draw a set of indifference curves for the following pairs of goods:  
 a) Printers and Ink Cartridges [2 marks]

**Labelled axis [1mark], IC [1mark]**

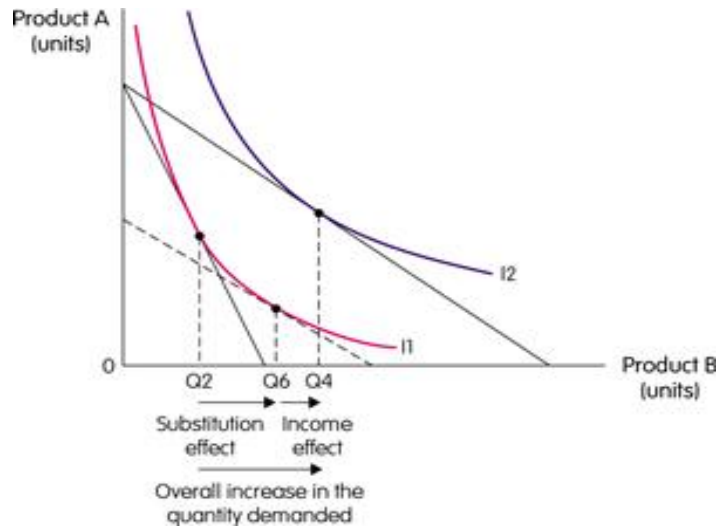


- b) The Wall Street and The Economist for an individual who regards these two magazines are perfect substitutes. [2 marks]

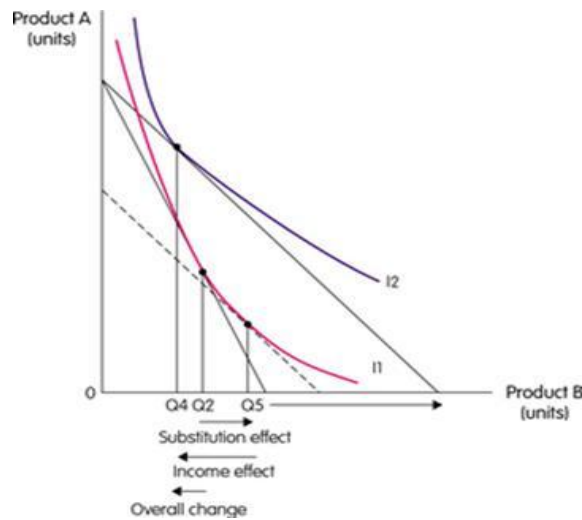
**Labelled Axis [1mark],**



- vi. indifference curve analysis, explain the difference between the income effect and the substitution effect of a fall in price for:  
 a. A normal good [4 marks]



Fall in price shown by adjusting budget line to the right [1mark], the substitution effect shown [1mark], compensated budget line shown [1mark] income effect shown [1mark]



b. An inferior good [4 marks]

Fall in price shown by adjusting budget line to the right [1mark], the substitution effect shown [1mark], compensated budget line shown [1mark] income effect shown [1mark]

vii. A demand function for a necessity is given as;  $Q = 10 - 2P$ . If the price elasticity of demand for this necessity at price  $P^*$  is  $-2/3$ . Find  $P^*$  [2marks]

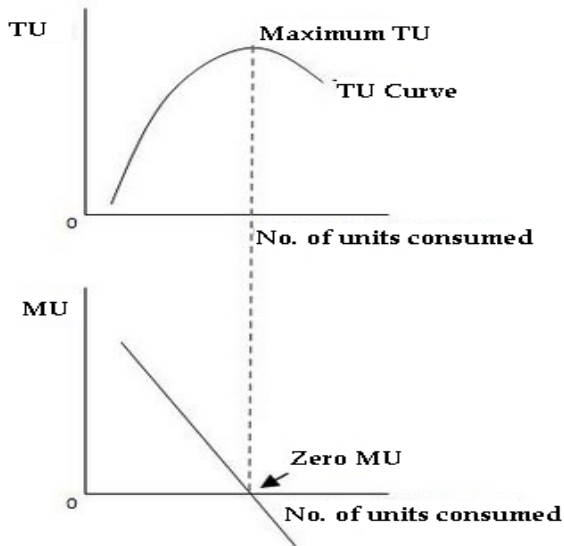
$P^* = 3$  [M1, A1]

viii. Outline **two** main determinants of the price elasticity of demand for a commodity

1. Availability of substitutes

2. Level of income spent on the commodity
3. Nature of the commodity. Necessities v/s luxuries
4. Breadth of definition of a good [@1/2 mark]

ix. Using a well labeled diagram, explain the law of diminishing marginal utility [2marks]



Award [1 mark] for the total utility function showing maximum utility and [1 mark] for the Marginal utility showing that MU becomes zero when TU reaches maximum point function

#### QUESTION FOUR

In Kenya, trade in Counterfeit Drugs, Marijuana, Cocaine and Heroine is strictly prohibited. However, people still trade in the black market. In the market, the demand for Counterfeit drugs is  $Q_d = 100 - 6P$ , and the supply of the Counterfeit drugs is  $Q_s = 60 + 2P$

[Total 30 marks]

- i. Find the equilibrium price and quantity in the black market [3 marks].

**Equating  $Q_d$  to  $Q_s$ ,  $P_e = 5$ ,  $Q_e = 70$  (@ 1mark)**

- ii. Find the consumer surplus[2 marks]

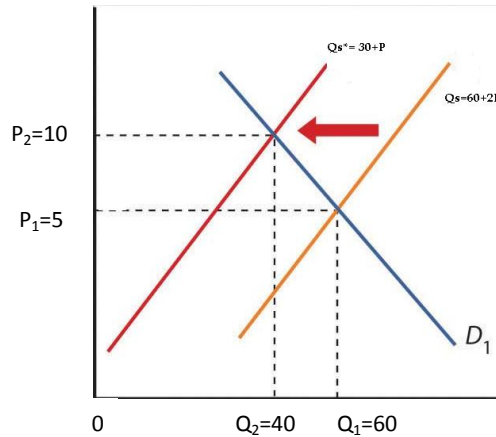
**Consumer surplus =  $\frac{1}{2} * 15 * 70 = 525$  (M1, A1)**

- iii. If the government puts in place stricter but costly monitoring measures and succeeds in seizing and destroying the counterfeit drugs thereby reducing their supply by half.

- a. Give the new demand and supply functions for the market[4 marks]

**$Q_d = 100 - 6P$ ,  $Q_s = 30 + P$  (@ 1 mark)**

- b. Find the new equilibrium price and quantity [3 marks]  
**Equating  $Q_d$  to  $Q_s$ ,  $P_e = 10$ ,  $Q_e = 40$  (@ 1 mark)**
- c. Find the change in consumer surplus following this intervention [3 marks]  
**Consumer Surplus after intervention =  $\frac{1}{2} * 40 * 6.67 = 133.40$  {M1, A1}**  
**The change in consumer surplus is given by  $[525 - 133.40] = 391.6$  (A1)**
- d. Show the change by using a supply and demand diagram [2 marks].



**Initial equilibrium shown (1 mark), new equilibrium after intervention shown (1 mark)**

- iv. Suppose the government is overwhelmed and so enacts legislation to allow open market trade in the counterfeit drugs. However, for every unit of counterfeit drugs purchased, the buyer has to pay tax,  $T$  to the government where  $T$  is equal to the pre-tax price  $P$  so that the price doubles.
- What will be the resultant demand and supply functions [4 marks]  
 **$Q_d = 100 - 6[P+T]$ ,  $Q_d = 100 - 12P$  [2 marks]**  
 **$Q_s = 30 + 2P$  [2 marks]**
  - Give the equilibrium (pre-tax) price and quantity [3 marks]  
 **$P_t = 5$  and  $Q_t = 40$  [M1, A1, A1]**
  - Find the after-tax price paid by buyers [2 marks]  
**The after tax price is 10 [Award 2 marks]**
- v. Compare policy choices (iii) and (iv). Explain the policy choice that would be favourable to;
- The Consumers [2 marks]  
**The consumers would be indifferent between the two policy choices because remain at the same price and quantity levels  $P_e = 10$   $Q_e = 40$  [2 marks]**
  - The Government [2 marks]



Policy choice two would generate revenue to the government and would be preferred  
[2marks]

### QUESTION FIVE

(a). A monopoly faces market demand  $Q = 30 - P$  and has a cost function  $C(q) = \frac{1}{2} q^2$

i. Find the monopolist's profit maximizing price, quantity; and the resulting profit

$$P = 30 - Q$$

$$TR = P \cdot Q = 30Q - Q^2 \quad [1 \text{ mark}]$$

$$\pi = TR - TC = 30Q - Q^2 - \frac{1}{2}Q^2$$

$$= 30Q - 1.5 Q^2 \quad [1 \text{ mark}]$$

$$\frac{\partial \pi}{\partial Q} = 30 - 3Q$$

$$30 - 3Q = 0, Q = 10 \quad [1 \text{ mark}]$$

$$P = 30 - Q = 30 - 10 = 20, \quad P = 20$$

$$\pi [\text{Profit}] = 30(20) - 1.5(20)^2 = 200$$

ii. Find the price elasticity of demand;  $E_D$  at the price and quantity found in (a). [2 marks]

$$E_D = \frac{\partial P}{\partial Q} \cdot \frac{Q}{P}$$

$$\frac{\partial P}{\partial Q} = -1, E_D = -1 \cdot \frac{10}{20} = -1/2 \text{ or } -0.5$$

iii. Calculate the consumer surplus. Show the consumer surplus on a diagram. [2 marks]

$$CS = \frac{1}{2} \cdot 10 \cdot 10 = 10 \quad [1 \text{ mark}] \text{ sketch/diagram } [1 \text{ mark}]$$

iv. Assume that the government puts a price ceiling on the monopolist at  $P = 18$ . How much output will the monopolist produce? What will be the profit of the monopolist? Calculate the change in consumer surplus [3 marks]

$$P = 30 - Q, Q = 12 \quad [1 \text{ mark}], \pi = 30(12) - 1.5(12)^2 = 144$$

(b). Let output be a function of two inputs

$$Q = AK^\alpha L^\beta$$

$$A(\lambda K)^\alpha (\lambda L)^\beta = A(\lambda^\alpha K^\alpha)(\lambda^\beta L^\beta) = \lambda^{\alpha+\beta} (AK^\alpha L^\beta) = \lambda^{\alpha+\beta} \cdot Q,$$

Scaling each factor input by  $\lambda$  increases output by  $\lambda^{\alpha+\beta}$ , meaning that this function is not homogeneous [1 mark]

i. Under what conditions would there be constant returns to scale? Increasing returns to scale?  
Decreasing returns to scale? [1 mark]

$\alpha + \beta > 1$  ICRTS

$\alpha + \beta = 1$  CRTS

$\alpha + \beta < 1$  DRTS

- ii. Find the  $MPP_K$  and the  $MPP_L$

[2 marks]

$$MPP_K = \partial Q / \partial K = \alpha AK^{\alpha-1} L^\beta = \alpha AK^\alpha L^\beta / K = \alpha Q / K = \alpha AP_K \text{ [1 mark]}$$

$$\text{Similarly, } MPP_L = \partial Q / \partial L = \beta AK^\alpha L^{\beta-1} = \beta AP_L \text{ [1 mark]}$$

[Total 15 marks]

### QUESTION SIX

In Kenya trade in Counterfeit Drugs is strictly forbidden, so people trade in a black market. Given that the demand for counterfeit drugs is  $Q_d = 12 - P$ , and its supply is  $Q_s = 2P$ .

- a) Find the equilibrium price and quantity in the black market [2 marks].

$$P = 4 \quad Q = 8$$

(M1, A1)

- b) The government becomes aware of the black market and reinforces the police so that half of the supply of counterfeit drugs would be seized and destroyed. Under this circumstance, what are the demand and supply functions? What is the new equilibrium price and quantity? Show the change by using a supply and demand diagram [3 marks].

$$Q_d = 12 - P, \quad Q_s = P, \quad P = 6, \quad Q = 6$$

M1, A1, A1

- c) How does the consumer surplus change between (a) and (b)? [1 mark]

In (a), the consumer surplus is  $\frac{1}{2} \times 8 \times 8 = 32$  and in (b), the consumer surplus is  $\frac{1}{2} \times 6 \times 6 = 18$

The consumer surplus decrease by;  $32 - 18 = 14$

M1, A1

- d) Suppose that the government changes the policy and legalizes cigarette trade. Now counterfeit drugs are traded in an open market. However, for every unit of counterfeit drugs purchased, the buyer has to pay tax  $T$  to the government.  $T$  is equal to the pre-tax price  $P$ . What are the demand and supply functions under this circumstance? What are the equilibrium (pre-tax) price and quantity? What is the after-tax price paid by buyers

[3 marks]

$$Q_d = 12 - (P+T) = 12 - 2P, \quad Q_s = 2P, \quad P = 3 \quad Q = 6. \text{ The after-tax price is } 2 \times 3 = 6$$

M1, A1,

A1

- e) Compare (b) and (d). Which policy do consumers prefer? Which policy does the government prefer? Why? [2 marks]

**The quantities and the (after-tax) prices paid by the buyers are the same in both. Consumers' are indifferent. A1**

**The policy in (b) has an expenditure outlay for the government while that in (d) brings in revenue. Therefore the government prefers (d) A1**

### QUESTION SEVEN

Suppose the market demand and market supply for a commodity are given as follows:

$$Q_d = 5000 - 3p$$

$$Q_s = 1000 + p.$$

- a) Determine the equilibrium price and quantity for the market. [3marks]

**Qd = Qs, M1, P= 1000, A1 Qe = 3,000 A1**

- b) Suppose the maximum price is fixed at Kes 1200. Illustrate the effect of this price control on the supply and demand diagram. Is there a shortage? If so, what is the excess demand? [5marks]

**Qd= 1400, A1 Qs= 2200, A1, There is no shortage, the excess supply= 800 A1, Illustration on a demand diagram Qd; 1Mark, Qs; 1Mark**

- c) Calculate the change in consumer surplus, producer surplus and Dead Weight Loss of the maximum price legislation

**Solution: Compute the relevant areas**

- d) The popularity of brown 'chapatis' caused consumers' tastes to shift away from bread in a certain market. Show the effect of this shift on the market for blue band, which is used mainly when people eat toast. [2 marks]

**Correct illustration of an inward shift of the quantity demanded of blue band (2marks)**

**(Total 10 marks)**

### QUESTION EIGHT

1. John has an income of Kes 800 per week. He demands 14 units of good X per week when the price of good X is Kes 3.60 per unit.

- a) When the price of good X increased to Kes 3.80 per unit, John's demand for good X fell from 14 units to 12 units per week (ceteris paribus). Calculate John's price elasticity of demand for good X (to 2 decimal places). **(4marks)**
- b) When John's income fell to Kes 750 per week his demand for good X also fell from 14 units to 9 units per week (ceteris paribus). Calculate John's income elasticity of demand for good X (to 2 decimal places). **(4marks)**

**These questions can be answered using Point or Arc measures of elasticity.**

a).  $e_D = (12-14)/14 \div (3.8-3.6)/3.6 = -2/14 \div 0.2/3.6 = 0.1429/0.0556 = -2.57$

**M1**

**M1**

**M1**

**A1**

**OR**

(i)  $e_D = (12-14)/13 \div (3.8-3.6)/3.7 = -0.1538/0.0540 = -2.85$

**OR**

(ii)  $e_m = (9-14)/14 \div (750-800)/800 = -0.3571/0.0625 = -5.71$

**M1**

**M1**

**M1**

**A1**

**OR**

$e_m = (9-14)/11.5 \div (750-800)/775 = 0.4348/0.0645 = -6.74$

2. Mary's demand curve for food is  $Q = 10 - 2P$ . Her price elasticity of demand for food at price  $P^*$  equals  $-2/3$ . How much is  $P^*$ ? **(4 marks)**

$\frac{\partial Q}{\partial P} \times \frac{P^*}{Q} = -2 \times \frac{P^*}{10-2P^*} = -2/3$

**M1,**

**M1**

$3P^* = 10 - 2P^*$

**M1**

$P^* = 2$

**A1**

**(Total 10 marks)**

**QUESTION NINE**

Eric receives utility from days spent traveling on vacation domestically (D) and days spent traveling in a foreign country (F) as given by the utility  $U(D, F) = DF$ . The price of a day spent traveling domestically is \$160 and in a foreign country \$200. Eric's annual budget for traveling is \$8,000. **(Total 30 marks)**

- a. Find Eric's utility maximizing choice of days traveling domestically and in a foreign country. Find also his utility level from consuming that bundle.

(5 marks)

$$MRS = (\partial U / \partial D) / (\partial U / \partial F) = F/D$$

$$PD / PF = 160/200$$

$$F = 4D/5$$

$$160D + 200F = 8,000$$

$$D^* = 25 \quad F^* = 20$$

$$U(25, 20) = 500$$

- b. Suppose that the price of domestic traveling increases to \$250 per day. Calling his budget for traveling  $x$ , (suppose by now that it is unknown) find the demand for  $D$  and  $F$  under the new prices as a function of  $x$ . (6 marks)

$$MRS = (\partial U / \partial D) / (\partial U / \partial F) = F/D$$

$$PD / PF = 250/200$$

$$F = 5D/4$$

$$250D + 200F = x$$

$$D^* = x / 500$$

$$F^* = x / 400$$

- c. Find the income necessary to make Eric reach the same utility level as before the price change. (4marks)

$$U(D^*, F^*) = (x / 500) * (x / 400) = 500$$

$$\Rightarrow x = \sqrt{100,000,000} = 10,000$$

- d. Compute the quantities demanded with the new prices and the income you found in section c. (2marks)

$$D^* = 10,000/500 = 20 \quad F^* = 10,000 / 400 = 25$$

- e. Compute the quantities demanded with the new prices and the original income. (2marks)

$$D^{**} = 8,000 / 500 = 16 \quad F^{**} = 8,000 / 400 = 20$$

- f. Using your previous answers tell us what is the total change in quantity of  $D$  due to the price increase in  $PD$  that the consumer experiences and what part of that change is due to income or substitution effects. Give definitions of what income and substitution effects mean. (5 marks)

Initial bundle:  $D^* = 25$   $F^* = 20$

Final bundle:  $D^{**} = 16$   $F^{**} = 20$

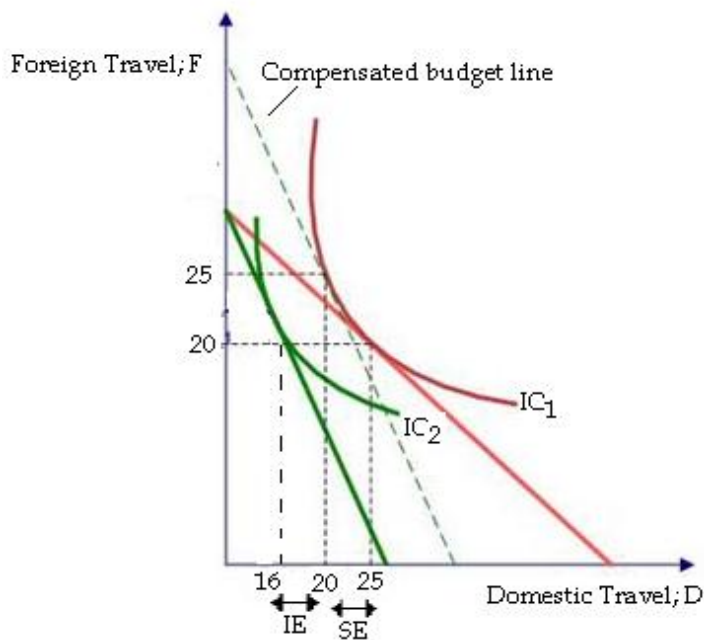
Compensated bundle:  $D^{c*} = 20$   $F^{c*} = 25$

Substitution effect  $20 - 25 = -5$  (1mark)

Income effect  $16 - 20 = -4$  (1mark)

We can decompose the change in quantity generated by a price change in two effects. The substitution effect is the variation in quantity generated by a change in the price ratio but holding utility constant (1mark). The income effect is the remaining change in quantity and is the result of the change in the purchasing power of the consumer (1mark).

g. Draw a graph showing the income and substitution effects you found (5marks).



3marks, (1 mark for @ correctly identified point), 1 mark for showing the substitution effect; 1 mark for showing the income effect

(Total 30 marks)

## QUESTION TEN

(a) Veronica's utility function is  $U(x,y) = x^2y$  with associated marginal utility functions of. She has an income ( $m$ ) of = \$ 240 and faces prices  $P_x = 8$  and  $P_y = 2$ . If the price of  $y$  increases to 8 and her income remains unchanged,

i. Calculate the numerical values of the substitution and income effects (8 marks)

$$P_x + P_y = M$$

$$8x + 2y_1 = 240 \dots (i)$$

At point of tangency  $MRS = MU_x/MU_y = P_x/P_y$  i.e. the ratio of marginal utilities is equal to the rate ratio of prices .

$$\text{For this case } 2xy/x^2 = 8/2$$

$$4xy = 8x^2$$

$$xy = x^2, y = x \dots (ii)$$

Solving the two simultaneous equations;

$$8x + 2y = 240$$

$$8x + 2x = 240$$

$$10x = 240, x = 24, y = 24$$

$$P_x + P_y = I$$

$$8x + 8y_2 = 240 \dots (i)$$

At point of tangency  $MRS = MU_x/MU_y = P_x/P_y$  i.e. the ratio of marginal utilities is equal to the rate ratio of prices.

$$\text{For this case } 2xy/x^2 = 8/8$$

$$x^2 = 2xy, x = 2y \dots (ii)$$

Solving the two simultaneous equations;

$$8x + 8y = 240$$

$$x + y = 30$$

$$2y + y = 30$$

$$3y = 30, y = 10, x = 20$$

At this point, the quantity of goods must satisfy the equation;

$$U_1(x,y) = x^2y = (24^2)24 = 13824 \dots (i)$$

Remember that at the point of tangency

$$MRS = MU_x/MU_y = P_x/P_y \text{ where } P_y = (\text{new price})$$

$$2xy/x^2 = 8/8$$

$$x = 2y \dots \text{(ii)}$$

Solving the two simultaneous equations;

$$x^2y = 13824$$

$$(2y)^2y = 13824$$

$$4y^2y = 13824$$

$$4y^3 = 13824$$

$$y^3 = 3456$$

$$y = 15, x = 30$$

Substitution effect is the effect of the movement from point A to B;

$$(B) - (A) = 15 - 24 = -9$$

Income effect is the effect of the movement from point B to C;

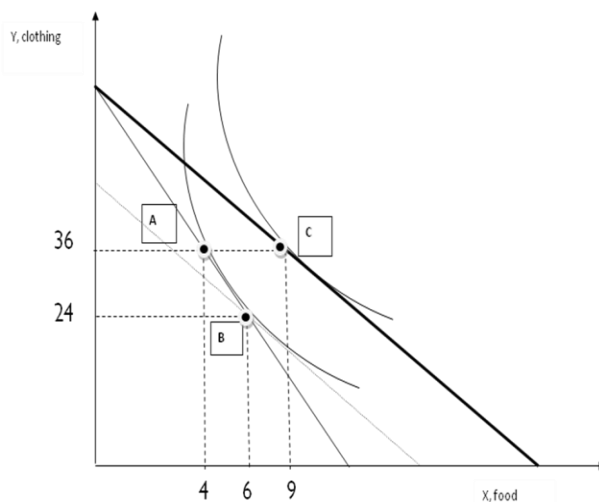
$$(C) - (B) = 10 - 15 = -5$$

ii. What is the meaning of income and substitution effect

**Substitution effect** refers to the change in demand due to the change in the rate of exchange between goods.

**Income effect** on the other hand refers to the change in demand due to change in the consumer's purchasing power.

iii. Illustrate the income and substitution effect of the price change on a graph





### QUESTION ELEVEN

(a) For each of the following production functions state the degree of homogeneity.

i.  $Q = AK^{1/4}L^{3/4}$

**Homogeneous of degree one**

ii.  $Q = AK^2L$

**Homogeneous of degree three**

iii.  $Q = AK^aL^b$

**Homogeneous of degree (a + b)**

(b) Compute the returns to scale for each of the following production functions

i.  $Q = AK^{\alpha}L^{1-\alpha}$

**Constant returns to scale**

ii.  $Q = AK^{1/4}L^{1/2}$

**Increasing returns to scale**

iii.  $Q = 2K^{0.5}L^{0.75} + K^{1.25} + L^{1.25}$

**Increasing returns to scale**

[Total 15 marks]

From here please complete the subsequent solutions

### QUESTION TWELVE

A monopoly faces market demand  $Q = 30 - P$  and has a cost function  $C(q) = \frac{1}{2}q^2$ .

i. Find the monopolist's profit maximizing price and quantity and the resulting profit

$$P = 30 - Q$$

$$TR = P \cdot Q = 30Q - Q^2 \quad [1 \text{ mark}]$$

$$\pi = TR - TC = 30Q - Q^2 - \frac{1}{2}Q^2$$

$$= 30Q - 1.5Q^2 \quad [1 \text{ mark}]$$

$$\frac{\partial \pi}{\partial Q} = 30 - 3Q$$

$$30 - 3Q = 0, Q = 10 \quad [1 \text{ mark}]$$

$$P = 30 - Q = 30 - 10 = 20, \quad P = 20$$

$$\pi [\text{Profit}] = 30(20) - 1.5(20)^2 = 200$$

[5 marks]

ii. Find the price elasticity of the demand  $E_D$  at the price and quantity found in (i) [4 marks]

$$E_D = \partial P / \partial Q * Q/P$$

$$\partial P / \partial Q = -1, E_D = -1 * 10/20 = -1/2 \text{ or } -0.5$$

- iii. What is the socially optimal price? Calculate the deadweight loss (**DWL**) due to the monopolist behavior of this firm. Calculate consumer surplus (**CS**) and producer surplus (**PS**). Show **CS**, **PS** and **DWL** on the diagram. **[5 marks]**

$$P = MC, P = 20$$

- iv. Assume that the government puts a price ceiling on the monopolist at  $P = 18$ . How much output will the monopolist produce? What will be the profit of the monopolist? Calculate **CS**, **PS** and **DWL**. Why is the deadweight loss different now? **[7 marks]**

- v. Assume that the government put a price ceiling on the monopolist in order to maximize the total (i.e. consumer plus producer) surplus. What price ceiling should it choose? How much output will the monopolist produce at this price ceiling? What will the profit of the monopolist be? What is the **DWL**? **[4 marks]**

- vi. Suppose the government charges the monopolist a \$130 fee for the right to operate. The monopolist doesn't pay the fee if it decides not to produce. Find the profit maximizing price and quantity and the resulting profits to the monopolist. Now assume that in addition to the \$130 fee, the government puts on the monopolist the same price ceiling as in part (e). Will the monopolist choose to produce in this case? **[5 marks]**

- vii. Suppose the government decides to impose a tax of \$3 per unit on the monopolist. Find the resulting output, price, government revenue and monopolist's profit. Show on a diagram the consumer surplus, the producer surplus, government revenue and deadweight loss.

**[3 marks]**

- viii. Assume the monopolist acquired a second factory with a cost function  $C_2(q) = 6q$ , but it still could use the first factory with the cost function  $C_1(q) = \frac{1}{2}q^2$ . Find the new profit maximizing price and calculate the amount of output produced in each of the two factories.

**[4 marks]**

**[Total 15 marks]**

### QUESTION THIRTEEN

The demand and total cost functions of some good are given by:

$$2P + 1/2Q - 8 = 0$$

$$C = 2 + Q - 3/20 Q^2 + 1/40 Q^3$$

### Compute

- i. The total revenue, TR, Profit;  $\pi$ , marginal revenue; MR and marginal cost; MC functions
- ii. At what value of  $Q^*$  is profit maximized
- iii. Compute the price charged at the quantity maximizing output  $Q^*$  in (b) above
- iv. Compute the profit realized
- v. Is the second order condition for profit maximization satisfied?

[Total 15 marks]

### QUESTION FOURTEEN

(a) Find the MRS  $x, y$  for the following utility functions

i.  $U = x^{1/3} y^{2/3}$

ii.  $U = x^{1/5} y^{4/5}$

iii.  $U = \sqrt{yx}$

(b). James consumes two goods  $x$  and  $y$ , his utility function is  $U(x, y) = 2xy^2$ . The price of  $x$  is \$10, while the price of  $y$  is \$15 and Ann's income is \$500. Suppose now the price of  $x$  is changed to 15, calculate the impact on Ann's optimum choice. Clearly showing the income and substitution effect of the price change?